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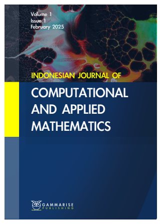

Research Article

Implementation of Fuzzy Time Series Markov Chain Method using Kernel Smoothing in forecasting the Stock Price of PT. Elnusa Tbk.

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Abstract

This research aims to apply the Fuzzy Time Series Markov Chain combined with Kernel Smoothing in forecasting stock prices. The Kernel Smoothing technique is used to smooth stock data before the fuzzification process, resulting in more accurate predictions. The research stages include Data Smoothing, Fuzzy interval formation, Fuzzy Logical Relationship and Fuzzy Logical Relationship Group formation, and forecasting using Markov Chain Transition Matrix. Evaluation using MAPE shows a low prediction error rate, with a value of 0.005974257%, so this method is effective for volatile stock data. The implementation of this model is expected to be a reference for investors and analysts in understanding and predicting future stock price movements.

Keywords : *Fuzzy Time Series Markov Chain · Kernel Smoothing · Share Price*
MSC2020 : *60J22 · 62A86 · 65C40 · 60G25*

1. Introduction

Indonesia's economic growth is shown by the rapid development of the capital market. This is supported by increasing public interest in investing, widespread public understanding of the capital market, and the increasing number of companies listed on the Indonesia Stock Exchange, which in turn encourages increased trading activity in the capital market. In a country's economy, the capital market has a strategic role as a means of raising funds for companies that require capital from the public (investors). Shares are one of the financial instruments that are the main choice of the public or investors when investing in the capital market. Before buying shares, investors will usually take into account the share price offered by the capital market. In the secondary market or in daily stock trading activities, stock prices fluctuate in the form of either an increase or a decrease. Stock prices that experience fluctuations in the form of both increases and decreases make investors need a model to see stock price movements [1].

With an indication of a spike or derivative in stock prices, it is necessary to establish a model in determining stock prices. One method that can be used in the forecasting process is the Fuzzy Time Series Markov Chain method. Fuzzy Time Series Markov Chain is used in price forecasting because it is a combination of the Fuzzy time series method with Markov chain. It aims to obtain the greatest probability in forecasting, as well as to determine the great opportunity by using the transition opportunity matrix.

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Tsaur first proposed a new concept of Fuzzy Time Series Markov Chain to analyze the accuracy in predicting the exchange rate of the Taiwanese currency against the US dollar. In his research, Tsaur combined the Fuzzy Time Series method with Markov chains. This combination aims to obtain the largest probability by using a transition probability matrix. Tsaur (2012) indicated that the Fuzzy Time Series Markov Chain Model method provides fairly accurate forecasting results. The use of prediction with the Fuzzy 4 Markov model has also been applied in the analysis of Taiwan's export and forex data by Wong and Wang [2]. The study confirms that the Markov model is effective for forecasting, while the fuzzy Markov model has a higher level of accuracy especially in longer forecasting periods [3].

There are several similar studies using the Fuzzy Time Series Markov chain method. These studies include Frianti et al. [4] predicting Crude Palm Oil (CPO) prices using the fuzzy time series markov chain method and in Dinatha et al. [5] research predicting export profits using the fuzzy time series markov chain method. In the research of Laily et al. [6] applying Fuzzy Time Series 3 Markov Chain to forecast rainfall as a rice crop schedule shows that the results of this method meet the criteria for excellent forecasting results because the MAPE value is less than 10 The results of this method show that the resulting value is more accurate. From the above studies, it can be concluded that the Fuzzy Time Series Markov Chain method is able to produce a high level of accuracy or quite accurate.

According to Härdle [7], the Kernel Smoothing method is a statistical technique used for smoothing data by utilizing kernel functions. This approach can be an option for processing volatile data such as stock prices. There are several types of kernel functions used, such as uniform kernel, triangle kernel, epanechnikov kernel, cosine kernel [8]. In this research, the case study used is the stock price of PT Elnusa Tbk, with variables such as Stock Price (Y) and time or day (X). The data will be analyzed using the software Rstudio.

2. Methods

2.1. Fuzzy Logic

Fuzzy logic is a form of logic that allows the truth value of a variable to exist between true and false values. This method allows the utilization of Fuzzy sets, which can be defined as curves with a triangular or trapezoidal shape, each of these curves has a slope where the value increases, a peak where the value equals 1, and a slope where the value decreases [9].

2.2. Time series

Time series is a set of data obtained over time at consistent intervals. Time series analysis is one of the statistical methods used to forecast future events. Time series analysis uses time-dependent data, so the correlation between current events and previous time periods becomes the main focus. Some methods applied in time series analysis include Moving Average (MA), Autoregressive (AR), and Autoregressive Integrated Moving Average (ARIMA) [10].

2.3. Fuzzy Time Series

The definitions related to forecasting Fuzzy Time Series are then described as follows [10].

Definition 1. [11] A fuzzy set is a collection of classes with different ranges of membership values. Suppose U is the universe of speech of U with its elements as $(u_1, u_2, u_3, \ldots, u_n)$, where u_1 is a possible linguistic value of U then a fuzzy set of linguistic variables of U is defined as follows.

$$A_i = \frac{\mu A_i(u_1)}{u_1} + \frac{\mu A_i(u_2)}{u_2} + \ldots + \frac{\mu A_i(u_n)}{u_n}, \tag{1}$$

where the membership function of the fuzzy set A_i is denoted by μ_{Ai}, so $\mu_{Ai} : U \to [0,1]$. If u_j is a membership of A_i then μ_{Ai} is the degree that u_j has on A_i

Definition 2. [12] Suppose $Y(t)(t = 0,1,2,...,n)$ is a subset of R, which is defined as a universe of speech with the set of fuzzy $fi(t)(i = 1,2,....,n)$ and $F(t)$ is a collection of $fi(t), f2(t),...,$ then $F(t)$ is called a fuzzy time series defined on $Y(t)(t = ...,1,2,...,n)$. Based on the concept, $F(t)$ can be considered as a linguistic variable. Since the value of $F(t)$ can vary at different times, $F(t)$, as a set of fuzzy *sets*, is a function of time t. Moreover, the Universe Discourse can be different at each time. Therefore, $Y(t)$ is used to represent the time t.

Definition 3. [11] If $F(t)$ is affected by $F(t-1)$ and indicated by $F(t-1) \to F(t)$, then there is a fuzzy relation between $F(t)$ and $F(t-1)$, which can be written by the formula

$$F(t) = F(t-1)o\Re(t,t-1), \tag{2}$$

where the relation R represents the model first order F(t) and 'o' is the max-min operator. If the fuzzy relation $\Re(t,t-1)$ of $F(t)$ is independent of time t (different times t_1 and t_2) then it can be written as $\Re(t1,t1-1) = \Re(t2,t2-1)$ so that $F(t)$ is called a time-invariant fuzzy time series.

Definition 4. [13] If $F(t) = A_i$ and $F(t-1) = A_j$, then the relationship between $F(t)$ and $(F(t-1)$ is known as a Fuzzy Logical Relationship (FLR). This relationship can be expressed by $A_i \to A_j$, where A_i is called the left-hand side (LHS) and A_i is called the right-hand side (RHS) of the FLR. Since two FLRs have the same fuzzy set $(LHSA_i \to A_{j1}, A_i \to A_{j2})$, they can be grouped into a fuzzy logical relationship group (FLRG) $A_i \to A_{j1}, A_{j2}$.

2.4. Fuzzy Time Series Markov Chain (FTSMC)

The fuzzy time series model Markov chain was first developed by a Russian expert named Andrey Andreyevich Markov in 1906. Markov chains are a mathematical method used to analyze the current behavior of some variables with the aim of predicting the behavior of the same variables in the future [14]. Markov chains are often illustrated by considering $(n = 0,1,2,...)$ as a mathematical process whose symptoms can be measured with an arbitrary degree of certainty, or the value of each probability can be calculated. The set of probabilities of this process is represented by the set of positive integers $(0,1,2,...)$. [15]. The following are forecasting steps using Fuzzy Time Series Markov Chain [16].

1. Collecting historical data and defining the universe of speech U.
   In this step, we find the minimum (D_{min}) and maximum (D_{max}) values of the historical data. After that, we determine the values of D_1 and D_2, which can be chosen freely and must be positive real numbers. The purpose of determining the values of D_1 and D_2 is to facilitate the formation of intervals. The formula for the universe of speech is

   $$U = [D_{min} - D_1, D_{max} + D_2], \tag{3}$$

   where

D_{min}	:	minimum value
D_{max}	:	maximum value
D_1, D_2	:	corresponding positive number value

2. Determine the number and length of fuzzy intervals.
   In this step, we partition the universe of speech U into several parts with the same interval (n) using the Sturges formula as follows.

   $$n = 1 + 3.322 \times \log N$$

   with n = many historical data. Next, calculate the interval length *(l)*.

   $$l = \frac{[(D_{max} + D_2) - (D_{min-D_1})]}{n} \tag{4}$$

   where

$$\begin{aligned} l &: \text{ interval length} \\ n &: \text{ many intervals} \end{aligned}$$

After that, the universe is divided into classes according to the number of intervals and their interval lengths

$$\begin{aligned} u_1 &= [D_{min} - D_1, D_{min} - D_1 + l] \\ u_2 &= [D_{min} - D_1 + 1, D_{min} - D_1 + 2l] \\ &\dots \\ u_n &= [D_{min} - D_1 + (n-1), D_{min} - D_1 + nl] \end{aligned} \tag{5}$$

Next, determine the middle value as follows.

$$m_i = \frac{\text{lower bound + upper bound}}{2}, \tag{6}$$

where i = many fuzzy sets

3. Define a fuzzy set on the universe of speech U.
   Defining the fuzzy set on the universe of speech U, the fuzzy set A_i denotes the linguistic variable $1 \leq i \leq n$. Determine the fuzzy set of the universe U. The entire fuzzy set can be determined as follows.
   $$A_i = \frac{f_{A_i(u_1)}}{u_1} + \frac{f_{A_i(u_2)}}{u_2} + \dots + \frac{f_{A_i(u_n)}}{u_n} \tag{7}$$
4. Fuzzyfication of historical data.
   Fuzzyfication is the process of identifying data into fuzzy sets. If a collected historical data falls within the interval u_i, then the data is fuzzified into A_i.
5. Determining Fuzzy Logical Relationship (FLR) and Fuzzy Logical Relation Group (FLRG).
   Relationships are identified based on a fuzzified value of historical data, as shown in the following example.
   $$\begin{aligned} LHS &\quad RHS \\ F(t-1) &\rightarrow F(t) \\ A_j &\rightarrow A_q \\ A_q &\rightarrow A_r \\ A_h &\rightarrow A_s \end{aligned} \tag{8}$$
   If the FLR "$A_j \rightarrow A_q$" is obtained, it can be interpreted that "if the data that has been fuzzified in year $t-1$ is A_j, then the result of fuzzifying the data in year t is A_q". Then, the FLRG is determined by combining the RHS that has an LHS. Based on the previous example, the FLRG obtained is as follows.
   $$\begin{aligned} A_j &\rightarrow A_q, A_r \\ A_h &\rightarrow A_s \end{aligned} \tag{9}$$
6. Create a Markov transition probability matrix.
   The FLRG obtained in the previous step can be used to find several possibilities from one state to the next. From these possibilities, a Markov probability transition matrix can be constructed with a transition matrix dimension of nxn. If state A_i moves to state A_j and passes through another state $A_k, i, j, k = 1, 2, \dots, n$, then FLRG can be obtained. The transition probability formula is as follows.
   $$P_{ij} = \frac{M_{ij}}{M_i}, i, j = 1, 2, \dots, n \tag{10}$$

Where, P_{ij}	=	transition probability from A_i to A_j with 1 step
M_{ij}	=	transition probability from state A_i to A_j with 1 step
M_i	=	the amount of data that state A_i has

The transition probability matrix R can be written as follows.

$$R = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \dots & & & \dots \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{bmatrix} \tag{11}$$

7. Calculating forecasting results (Forecasting Value).
The whole transition system is reflected by the matrix R. If $F(t-1) = A_i$, the process is defined to be the state A_i at time $t-1$, then the forecast for $F(t)$ will be calculated by using the row vector $[P_{i1}, P_{i2}, ..., P_{in}]$. The prediction result $F(t)$ is equal to the weighted average of $m_1, m_2, ..., m_n$. to determine the midpoint of the interval $u_i (i = 1, 2, ..., k)$. The value of the prediction output on $F(t)$ can be determined by following some rules.
Rule 1. If the Fuzzy Logical Relation Group (FLRG) of A_i has a one to one relationship i.e. ($A_i \to A_k$ with $P_{ik} = 1$ and $P_{ij} = 0, j \neq k$) then the forecast of $F(t)$ is m_k, the middle value of u_k.
$$F(t) = m_k P_k = m_k \tag{12}$$
Rule 2. If the Fuzzy Logical Relation Group (FLRG) of A_j is one to many which means ($A_j \to A_i, A_2, ..., A_n, j = 1, 2, ...n$) when the historical data $Y(t-1)$ at time $t-1$ is in state A_j then the forecast $F(t)$ will be equal to :
$$F(t) = m_1 P_{j1} + m_2 P_{i2} + m_{j-1} P_{j(j-1)} + Y_{t-1} P_{jj} + m_{j-1} P_{j(j+1)} + ... + m_n P_n, \tag{13}$$
where $m_1, m_2, ..., m_n$ is the mean of $u_1, u_2, ..., u_n$, Y(t-1) is the state value of A_i at time $t-1$.
8. Calculating the adjustment value on forecasting (Adjusted Value).
The purpose of this stage is to correct the forecasting error caused by bias in the Markov chain matrix. The bias in this matrix is often caused by the smaller sample size when modeling the Fuzzy Time Series Markov Chain model. Therefore, the following are the rules for calculating the adjustment value (D_t) in forecasting.
Rule 1. If state A_i corresponds to A_i, starting from state A_i at time $t-1$ as $F(t-1) = A_i$, and undergoing an increasing transition to state A_j at time $t, (i > j)$, then the value of D_t is
$$D_{t1} = \left(\frac{l}{2}\right) \tag{14}$$
Rule 2. If state A_i corresponds to A_i, starting from state A_i at time $t-1$ as $F(t-1) = A_i$, and undergoing a decreasing transition to state A_j at time $t, (i > j)$, then the value of D_t is
$$D_{t1} = -\left(\frac{l}{2}\right) \tag{15}$$
Rule 3. If the transition starts from state A_i at time $t-1$, as $F(t-1) = A_i$, and undergoes a forward transition to state $A_j + s$ at time $(t, 1 \leq s \leq n-1)$, then the value of D_t is
$$D_{t2} = \left(\frac{l}{2}\right) s, (1 \leq s \leq n-i), \tag{16}$$
where s = number of forward jumps.

Rule 4. If the transition starts from state A_i at time $t-1$, as $F(t-1) = A_i$, and undergoes a backward transition to state $A_j + s$ at time $(t, 1 \leq v \leq n-1)$, then the value of D_t is

$$D_{t2} = \left(\frac{l}{2}\right) v \left(1 \leq v \leq i\right), \tag{17}$$

where v = number of backward jumps.

9. Determining a suitable forecast result (Adjusted Forecasting Value).
   (a) If fuzzy logical relationship group A_i is one to many and state A_{i+1} is accessible from A_i where state A_i interacts with A_i then the forecasting result will be

$$F'(t) = F(t) + D_{t1} + D_{t2} = F(t) + \frac{l}{2} + \frac{l}{2} \tag{18}$$

where

$F'(t)$	:	forecasting value at the end of period $to - t$
$F(t)$	:	initial forecasting value in period t
$D(t)$	:	adjustment value

   (b) If FLRG A_i is one to many and state A_{i+1} is reachable from A_i where state A_i does not communicate with A_i then the forecasting result becomes

$$F'(t) = F(t) + D_{t2} = F(t) + \frac{l}{2} \tag{19}$$

   (c) If FLRG A_i is one to many and state A_{i+2} is reachable from A_i where state A_i does not communicate with A_i then the forecasting result becomes

$$F'(t) = F(t) - D_{t2} = F(t) - \frac{l}{2} \times 2 = F(t) - l \tag{20}$$

   (d) When v is a jump step, the forecasting result becomes

$$F'(t) = F(t) \pm D_{t1} \pm D_{t2} = F(t) \pm \frac{l}{2} \pm \frac{l}{2} v \tag{21}$$

10. Calculating the MAPE value
    The formula for MAPE is as follows

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \frac{|Y(t) - F'(t)|}{Y(t)} \times 100\%, \tag{22}$$

	$Y(t)$	:	actual value to-t
where	$F'(t)$	:	the final forecasting value in period t
	n	:	number of samples

11. Kernel Smoothing
    The commonly chosen kernel is the standard normal distribution kernel $K(z) = \frac{1}{\sqrt{2\pi}} exp \frac{-z^2}{2}$. Then the weights for y_i are given by [8]

$$w_{ij(ker)} = \frac{K\left(\frac{x-x_i}{h}\right)}{\sum_{j=1}^{n} K\left(\frac{x-x_i}{h}\right)} y_i. \tag{23}$$

    It can be written as

$$\hat{f}(x) \sum_{j=1}^{n} \frac{K\left(\frac{x-x_i}{h}\right)}{\sum_{j=1}^{n} K\left(\frac{x-x_i}{h}\right)} y_i. \tag{24}$$

Table 1. Data Smoothing Results

Period	Kernel Smoothing Results
January 2021	370.1396
February 2021	378.9190
⋮	⋮
October 2023	424.2828
November 2023	413.3909
December 2023	396.6978

3. Results and Discussion

3.1. Data smoothing using (Kernel Smoothing)

In this study, a normal Gaussian kernel with a bandwidth of 2 is used, which determines the width of the smoothing window. Here are the results of Kernel Smoothing :

3.2. Fuzzy Time Series Markov Chain

1. Defining the Talking Universe U From the data, the minimum value (D_{min} = 263.6494) and maximum value (D_{max} = 424.2828) are obtained. D_1 = 0.2 and D_2 = 0.3 where the values of D_1 and D_2 are positive random numbers determined by the researcher. After obtaining the values of D_{min}, D_{max}, D_1 and D_2, then the universe of speech U can be defined as follows.

$$U = [D_{min} - D_1, D_{max} + D_2]$$
$$U = [263.6494 - 0.2, 424.2828 + 0.3]$$
$$U = [263.4494; 424.5828]$$

2. Determining the Number and Length of Fuzzy Intervals
The following equation is used to determine the number of intervals.

$$n = 1 + 3.322 \times \log N$$
$$n = 1 + 3.222 \times \log(36)$$
$$n = 6$$

Next, find the length of the interval by using the value of l, which is obtained as follows.

$$l = \frac{[(D_{max} + D_2) - (D_{min} - D_1)]}{n}$$
$$l = \frac{[424.2828 + 0.3) - (263.6494 - 0.2)]}{6}$$
$$l = \frac{[424.5828 - 263.4494]}{6}$$
$$l = 26.85557$$

After obtaining the interval length (l = 26.85557), the universe can be partitioned into six parts. Here are the intervals obtained.

$$u_1 = [D_{min} - D_1, D_{min} - D_1 + l]$$
$$u_1 = [263.6494 - 0.2, 263.6494 - 0.2 + 26.85557]$$
$$u_1 = [263.4494, 290.3049]$$

Next, determine the middle value (M)

$$m_i = \frac{\text{lower bound} + \text{upper bound}}{2}$$
$$m_1 = \frac{263.4494 + 290.3049}{2}$$
$$m_1 = 276.8772$$

Table 2. Division of the Talking Universe

Interval	Lower Bound	Upper Bound	Mean Value
U_1	263.4494	290.3049	276.8772
U_2	290.3049	317.1605	303.7327
U_3	317.1605	344.0161	330.5883
U_4	344.0161	370.8717	357.4439
U_5	370.8717	397.7272	384.2994
U_6	397.7272	424.5828	411.1550

3. Defining fuzzy sets in the universe of speech U
   Here are six fuzzy sets formed based on the number of intervals U.

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6}$$
$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6}$$
$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6}$$
$$A_4 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{0.5}{u_5} + \frac{0}{u_6}$$
$$A_5 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0.5}{u_4} + \frac{1}{u_5} + \frac{0.5}{u_6}$$
$$A_6 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0.5}{u_5} + \frac{1}{u_6}$$

   After the application of the membership degree u_i to the fuzzy set A_i is carried out, the interval fuzzy set and its fuzzification are obtained at Table 3.

Table 3. Interval fuzzy sets and their fuzzification

Interval	Fuzzification
u_1 = [263.4494, 290.3049]	A_1
u_2 = [290.3049, 317.1605]	A_2
u_3 = [317.1605, 344.0161]	A_3
u_4 = [344.0161, 370.8717]	A_4
u_5 = [370.8717, 397.7272]	A_5
u_6 = [397.7272, 424.5828]	A_6

   Based on Table 3, the fuzzification result of the Share Price of PT Elnusa Tbk. in the October 2023 period is A_6. This is because the Share Price of PT Elnusa Tbk. for the October 2023 period has a nominal value of 424.2828. The nominal is in the interval range u_6, which is the interval with the maximum membership degree in defining the membership degree *fuzzy set* A_6.
4. Fuzzification of historical data
   The following are the results of data that has been fuzzified.

Table 4. Fuzzyfication Result

Period	Data	Fuzzyfication
1	370.1396	4
2	378.9190	5
3	358.8020	4
⋮	⋮	⋮
34	424.2828	6
35	413.3909	6
36	396.6978	5

5. Define *Fuzzy Logical Relationship* (FLR))
   The overall FLR of PT. Elnusa Stock Price data is as follows.

Table 5. *Fuzzy Logical Relationship* (FLR)

Data Sequence	FLR
$1-2$	$4 \to 5$
$2-3$	$5 \to 4$
$3-4$	$4 \to 3$
⋮	⋮
$33-34$	$6 \to 6$
$34-35$	$6 \to 6$
$35-36$	$6 \to 5$

6. Forming Fuzzy Logical Relationship Groups (FLRG)
   The results of the formation of Fuzzy Logical Relationship Groups (FLRG) can be seen in Table 6.

Table 6. *Fuzzy Logical Relationship Group* (FLRG)

Current State	Next State
A_1	$4(A_1)$, $2(A_2)$
A_2	$2(A_1)$, $10(A_2)$, $2(A_3)$
A_3	$2(A_2)$, A_3, $2(A_4)$
A_4	$2(A_2)$, $2\ (A_5)$
A_5	(A_4), A_5, A_6
A_6	A_5, $2(A_6)$

7. Forming the Markov Transition Probability Matrix
   Here is the Markov Transition Probability Matrix.

$$P_{ij} = \frac{M_{ij}}{M_i}$$
$$P_{11} = \frac{4}{6}$$
$$P_{11} = 0.66667$$

The results of forming another Markov transition probability matrix following the same steps are as follows.

$$R = \begin{bmatrix} 0.66667 & 0.33333 & 0 & 0 & 0 & 0 \\ 0.14286 & 0.71429 & 0.14286 & 0 & 0 & 0 \\ 0 & 0.40000 & 0.20000 & 0.40000 & 0 & 0 \\ 0 & 0 & 0.50000 & 0 & 0.50000 & 0 \\ 0 & 0 & 0 & 0.33333 & 0.33333 & 0.33333 \\ 0 & 0 & 0 & 0 & 0.33333 & 0.66667 \end{bmatrix}$$

8. Calculating Forecasting Results(Forecasting Value)
After obtaining the Markov transition probability matrix, forecasting is then carried out. For example, the calculation of forecasting the Share Price of PT Elnusa for the March 2021 period as $F(t)$, the Share Price of PT Elnusa when $(t-1)$ is in state A_5, and FLRG A_5 has a one-to-many relationship $(A_5 \rightarrow A_4, A_5, A_6)$, then the results of forecasting the Share Price of PT Elnusa for the March 2021 period are as follows.

$$F(t) = m_4 P_{54} + Y(t-1)P_{55} + m_6 P_{56}$$
$$F(t) = (357.4439)(0.33333) + (378.9190)(0.33333) + (411.1550)(0.33333)$$
$$F(t) = [382.5021]$$

9. Calculate the adjustment value of forecasting (Adjusted Value)
Based on the Markov Transition Probability Matrix that has been formed, it is known that state A_4 and state A_5 communicate with each other, and there is an upward transition from state A_4 to state A_5. Thus, the adjustment of the forecasting value of the Share Price of PT Elnusa for the period January 2021 is determined as follows.

$$D_t = \frac{l}{2}$$
$$D_t = \frac{26.85557}{2}$$
$$D_t = 13.42779$$

10. Determine the Final Forecasting Result
The final results of forecasting the Stock Price of PT Elnusa can be seen in Table 7.

Table 7. Final Forecasting Results

Period	Final Forecasting Results
January 2021	370.8717
February 2021	369.0743
March 2021	344.0161
⋮	⋮
September 2023	404.4612
October 2023	410.9551
November 2023	390.2660

3.3. Calculating the MAPE Value (*Mean Absolute Percentage Error*)

Calculating the MAPE value is done to measure the level of accuracy by identifying the absolute error in each period by dividing the observation value in that period and then converting it into a percent value. The calculation results show a MAPE value of 0.005974257%.

4. Conclusion

Based on the results of the discussion previously described, it can be concluded that the results of forecasting the stock price of PT Elnusa Tbk using the Fuzzy Time Series Markov chain combined with Kernel Smoothing show that this method is effective in forecasting the stock price of PT Elnusa for the period January 2021 to December 2023. The use of Kernel Smoothing in this model successfully reduces the uncertainty of stock price data and provides smoother and more accurate forecasting results. In addition, the level of accuracy obtained shows that this method is able to produce quite good and reliable forecasts for the movement of the share price of PT Elnusa Tbk. These results indicate that the Fuzzy Time Series Markov Chain method using Kernel Smoothing has the potential to be used as a tool in capital market analysis, especially in supporting the investment decision-making process.

Supplementary Information

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Conflict of interest. The authors declare no conflict of interest.

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