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Research Article

Application of the Monte Carlo Method to Pricing Lookback Fixed Option with Stochastic Volatility

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Abstract
Options are a derivative product that trades the right to call and put on an asset at a certain price and during an agreed time. Determining the optimal option price is often difficult due to changes in stock prices. One model that can be used to calculate the price of Lookback Fixed options is the Monte Carlo Method with stochastic volatility of the Heston model, with parameter estimation using Ordinary Least Squares (OLS), and Euer-Maruyama and calculation of the effect of initial stock price, strike, and maturity time. The estimated stock price is then used to calculate the Lookback Fixed option price using the Monte Carlo method. The research results obtained good results with a fairly small error rate. In addition, the analysis of the effect of strike price, strike, and maturity time shows results consistent with option pricing theory.

Keywords : *Option Pricing · Lookback Fixed Options · Heston Volatility Model · Monte Carlo Method*
MSC2020 : *91G60 · 91G30 · 91G20 · 65C05*

1. Introduction

Options are derivative instruments commonly used by investors in stock trading. They grant the holder the right to purchase (call) or sell (put) an asset at a specified price within a designated timeframe[1, 2]. In addition to standard options (vanilla options), there are also exotic options, whose value is determined by the movement of the asset's value during the option's term [3]. One type of exotic option is the lookback option, which bases its pricing on the highest or lowest price of the underlying asset during the option period [4–10].

Lookback options are divided into two types based on the strike price determination: floating strike and fixed strike. In a lookback option with a fixed strike, the strike price remains constant throughout the option's life and does not change from the value specified at the contract's start. One of the challenges in options trading is that stock price fluctuations significantly impact options valuation [11]. Therefore, stochastic volatility models, such as the Heston model, are often used to account for varying volatility [12–15]. This model does not have an analytical solution, so numerical methods, such as Monte Carlo simulations, are used to approximate the solution[16–19]. This method's advantage is that the closer the estimated results are to the analytical solution, the more simulations are performed. In other words, the Monte Carlo method tends to converge to the analytical solution as the number of simulations increases [20–22]. Research on stochastic volatility models using the Monte Carlo method has been conducted by Chalimatusadiah et al., focusing on the optimal pricing of European-type options through a Monte Carlo method approach with stochastic volatility [23].

The purpose of this study is to simulate the Lookback fixed strike option price for PT Bank Syariah Indonesia Tbk (BSI) shares in 2022-2023 using the Heston stochastic volatility model and the

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Monte Carlo method and analyze the factors that can affect the option price.

2. Methods

The method used is a literature review of relevant books and journals on applying the Monte Carlo simulation method in pricing Lookback Fixed options with stochastic volatility. The secondary data consists of the closing price and weekly volatility of Bank Syariah Indonesia (BSI) shares for the 2022-2023 [24]. The following steps are provided.

1. Calculate the logarithmic return from the weekly stock price data.
2. Test the normality of the logarithmic stock returns.
3. Estimate the parameters of the Heston volatility model using the Euler-Maruyama numerical method and Ordinary Least Squares (OLS).
4. Calculate the price of the Fixed Lookback option using the Monte Carlo method.
5. Analysis of factors affecting the price of Lookback options.

2.1. Algorithm for estimating model parameter values

The following is the algorithm for estimating the parameters of the Heston volatility model using the Euler-Maruyama method and Ordinary Least Squares (OLS) with Scilab 6.0.2.

1. Input the stock price, volatility, time interval (Δt), maturity time (T), and number of simulations (M).
2. Determine the parameter values of the Heston model, which is assumed to follow a log-normal distribution for S_t and a Cox-Ingersoll-Ross process for V_t. The model is given as follows.

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{V_t} S_t dW_t \\ dV_t &= \kappa(\theta - V_t)dt + \eta\sqrt{V_t} dZ_t \\ dW_t dZ_t &= \rho dt \end{aligned} \tag{1}$$

The estimation of the parameters $\kappa, \theta, \sigma, \mu$ and ρ is done using the Euler-Maruyama method, Ordinary Least Squares (OLS), and several formulas to calculate the standard deviation and correlation coefficient. The parameters κ and θ are estimated as follows.

$$\kappa = \frac{1-h}{\Delta t}, \ \theta = \frac{g}{1-h}.$$

with

$$g = \frac{\sum_{i=1}^{N-1}\left(\frac{V_{i+\Delta t}}{\eta^2 V_i \Delta t}\right) - h\sum_{i=1}^{N-1}\left(\frac{1}{\eta^2 \Delta t}\right)}{{}_{i=1}^{n-1}\left(\frac{1}{\eta^2 V_i \Delta t}\right)}$$

$$h = \frac{\sum_{i=1}^{N-1}\left(\frac{V_{i+\Delta t}}{\eta^2 \Delta t}\right) - \frac{\frac{(N-1)}{\eta^2\Delta t}\sum_{i=1}^{N-1}\left(\frac{V_{i+\Delta t}}{V_i}\right)}{\sum_{i=1}^{N-1}\left(\frac{1}{V_i}\right)}}{\sum_{i=1}^{N-1}\left(\frac{V_i}{\eta^2\Delta t}\right)\frac{\frac{(N-1)^2}{\eta^2\Delta t}}{\sum_{i=1}^{N-1}\left(\frac{1}{V_i}\right)}}$$

The parameter σ is estimated as follows.

$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(u_i - \bar{u})^2} \tag{2}$$

Where $\bar{u}$ is the mean value of the log return, while the standard deviation of u_i is $\sigma\sqrt{\Delta t}$ so that s becomes an estimate of $\sigma\sqrt{\Delta t}$. Thus, the volatility of asset prices is obtained as follows.

$$\bar{\sigma} = \frac{s}{\sqrt{\Delta t}} \tag{3}$$

The parameter ρ is given by

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}. \tag{4}$$

where

r	:	Correlation coefficient
x_i	:	The value of the variable x at the i-th observation in the sample
y_i	:	The value of the variable y at the i-th observation in the sample
$\bar{x}$	:	The average of the x variables in the sample
$\bar{y}$	:	The average of the y variables in the sample

2.2. Algorithm for calculating the option price

The parameter estimation results are used to derive the price of the Lookback Fixed option by applying the Monte Carlo method. The option price calculation is performed with the help of Scilab 6.0.2 software using the following steps.

1. Enter the initial stock price (S_0) and volatility (V_0),
2. Determine the number of stock price subintervals (N) using the formula $N = \dfrac{T}{\Delta t}$
3. Calculate (S_i) for $i = 1,2,3,\ldots,M$ using the model.

$$\begin{aligned} dS_t &= \mu S_t\,dt + \sqrt{V_t}S_t\,dW_t, \\ dV_t &= \kappa(\theta - V_t)\,dt + \eta\sqrt{V_t}\,dZ_t. \end{aligned} \tag{5}$$

   The values (S_1) and (V_1) are used to calculate $S_2, S_3, S_4, \ldots, S_M$
4. Determine the payoff of the Lookback option in the $i-t$h simulation as follows.

$$A_i = \begin{cases} max(S_{max(i)} - K, 0); i = 1,2,3,...,n & \text{for call options,} \\ max(K - S_{min(i)}, 0); i = 1,2,3,...,n & \text{for put options.} \end{cases}$$

5. The Lookback Fixed option price in the $i-th$ simulation using the Monte Carlo method is

$$O_{MC}^{(i)} = e^{-rT}\mathbb{E}(A_i), \quad i = 1,2,3,\ldots,M, \tag{6}$$

   for r is the risk-free interest rate
6. The Monte Carlo estimator for estimating the Lookback Fixed option price (O) is:

$$\hat{O}_{MC} = \frac{1}{M}\sum_{i=1}^{M} O_{MC}^{(i)} \tag{7}$$

7. The value of the Lookback Fixed option is estimated as follows:

$$O \approx \hat{O}_{MC}$$

3. Results and Discussion

3.1. Logarithmic Return Calculation

In calculating the logarithmic return, we can use

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right), t = 1, 2, \ldots, n \tag{8}$$

where

R_t	:	Logarithmic return of the stock price at time t
S_t	:	stock price at time t
S_{t-1}	:	stock price at time $(t-1)$

The following graph shows the data from the calculation of the logarithmic returns.

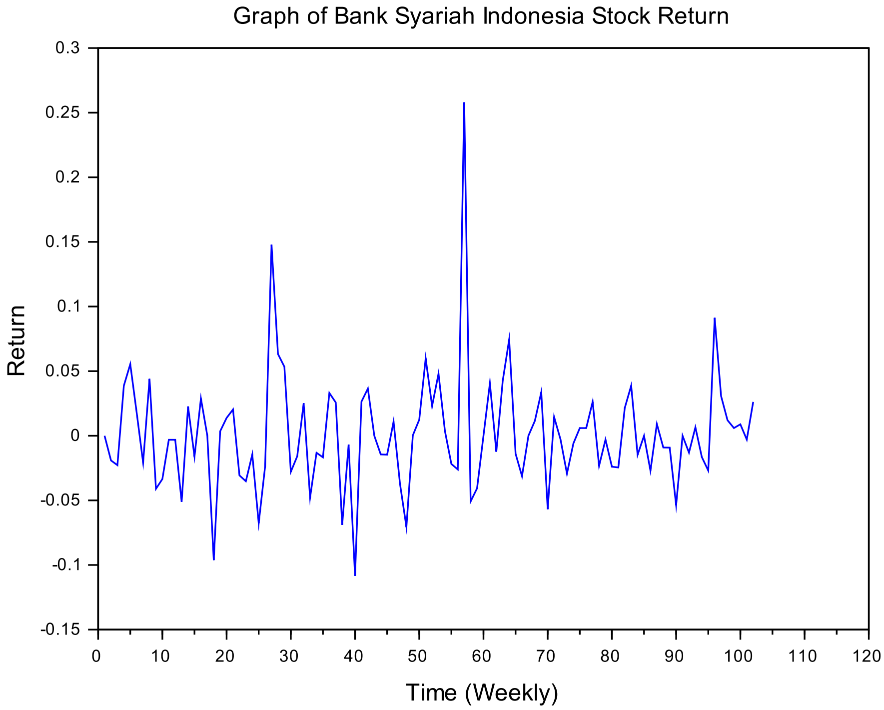

Figure 1. Effect of the strike price on the Lookback Fixed option price.

Based on Figure 1, we can see that the logarithmic return value fluctuates every week, increasing and decreasing, with 102 data points for the weekly stock returns.

3.2. Data Normality Test

The logarithmic stock return data that has been obtained is then tested for normality using the Kolmogorov-Smirnov test with RStudio software. From the test results, the average stock price return of Bank Syariah Indonesia (BSI) is 0.001171305, the standard deviation is 0.045534977, and the p-value is 0.89, with a confidence level (α) of 0.05. Since the p-value is more significant than > 0.05, we conclude that the stock price return of Bank Syariah Indonesia for the period 2022-2023 follows the normal distribution.

3.3. Numerical Results of Parameter Estimation for the Heston Volatility Model

The estimated values of the parameters κ, θ, η, and ρ are obtained by estimating the parameters of the Heston model in equation (1), with the help of Scilab 6.0.2, as follows.

Table 1. Estimated model parameter values

Parameter	Value
κ	26.14085
θ	0.02065
η	0.01419
ρ	0.0384

3.4. Numerical Results of Option Prices Using Monte Carlo Simulation

In the numerical simulation, the stock price is approximated using the parametric values, and the option price is then calculated using the Monte Carlo method. The parameters used include initial volatility $(V_0) = 20\%$, initial stock price $(S_0) = \$100$, time interval $(\Delta t) = \frac{1}{52}$ year, and the number of simulations (M). This process involves random number generation and is performed using Scilab 6.0.2 software. After obtaining the stock price from the simulation, the option price is calculated using input parameters such as maturity time $(T) = \frac{12}{52}$ years, strike price $(K) = \$100$, and risk-free interest rate $(r) = 2.45\%/\text{year}$.

Table 2 presents the value of the Fixed Lookback option calculated using the Monte Carlo Simulation Method with various selected values of M. It also includes the relative error for each simulation, which is determined using equation (11).

$$\varepsilon_r = \frac{|O^{(i)} - \hat{O}|}{\hat{O}} \tag{9}$$

Let $\hat{O}$ represent the approximate option price for the unknown option value, and $O^{(i)}$ represent the option price at the i-th simulation. We have the following results.

Table 2. Lookback Fixed option price simulation results and relative error

Simulation (M)	call option price ($)	put option price ($)	relative error of call options	relative error of put options
1000	6.88	6.45	0.030680	0.049862
2000	6.82	6.40	0.039882	0.041203
3000	6.86	6.39	0.034514	0.039293
4000	6.89	6.24	0.030017	0.014833
5000	6.89	6.36	0.030008	0.034560
6000	7.17	6.15	0.010104	0.000245
7000	6.94	6.29	0.023144	0.023539
8000	7.04	6.18	0.009137	0.005659
9000	7.12	6.13	0.002154	0.002438

In Table 2, analytical solutions are derived based on simulations conducted within the maximum memory capacity of the computer, with $M = 10000$. The Lookback Fixed Call option price is 7.10, while the Lookback Fixed Put option price is 6.15. It can be observed that as the number of simulations increases, the relative error decreases. The results are quite accurate, with the relatively small error at $M = 9000$: 0.002154 for call options and 0.002438 for put options.

3.5. Analysis of Factors Affecting Lookback Fixed Option Prices.

Several factors affect the price of an option, including the strike price, the initial stock price, and the time to maturity. This study analyzes the influence of various factors on the movement of call and put option values. The values of call and put options are evaluated with 2000 simulations using the Monte Carlo method.

In Figure 2, the price of call options tends to decrease as the strike price increases, while the price of put options tends to increase.

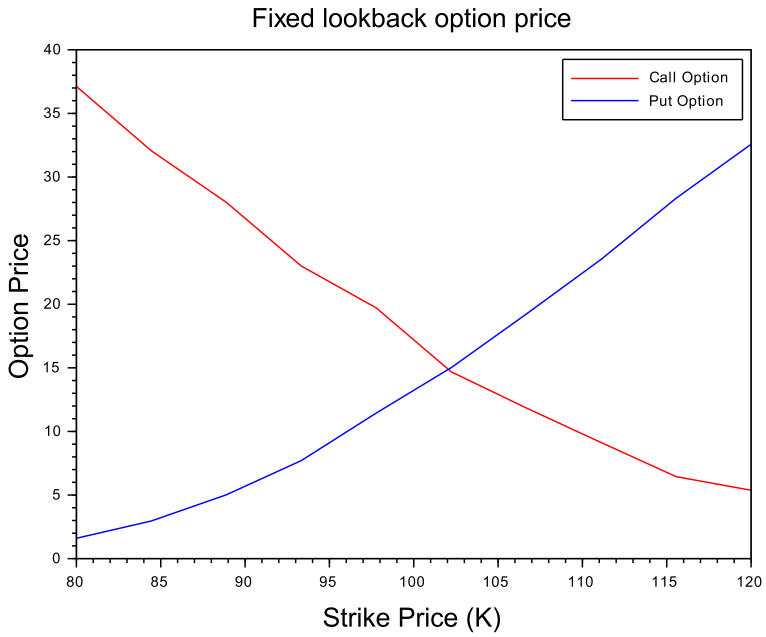

Figure 2. Effect of strike price on option price Lookback Fixed

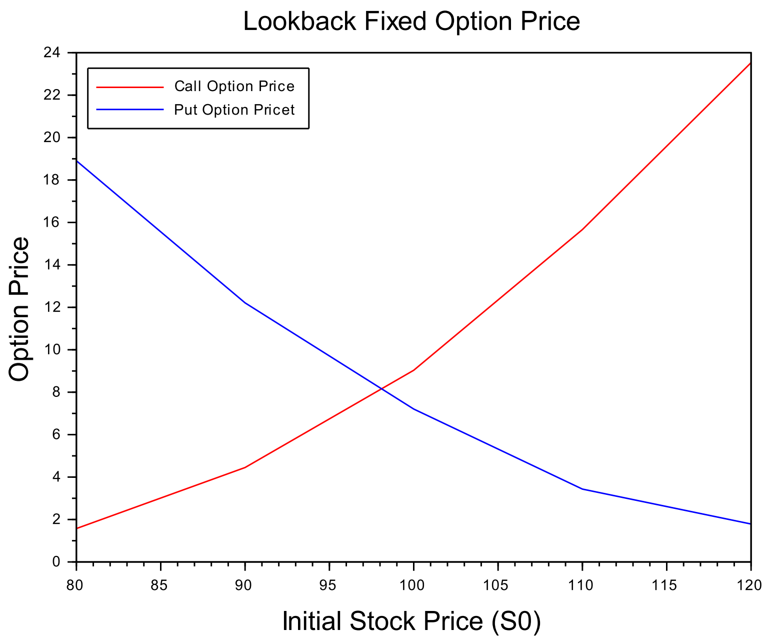

Figure 3. Effect of initial share price on option price Lookback Fixed

Figure 3 shows that the price of call options increases as the initial stock price increases, while the price of put options tends to decrease as the initial stock price increases.

Figure 4 shows that the increase in the value of call and put options is influenced by a longer maturity time, which provides more time for stock price movements.

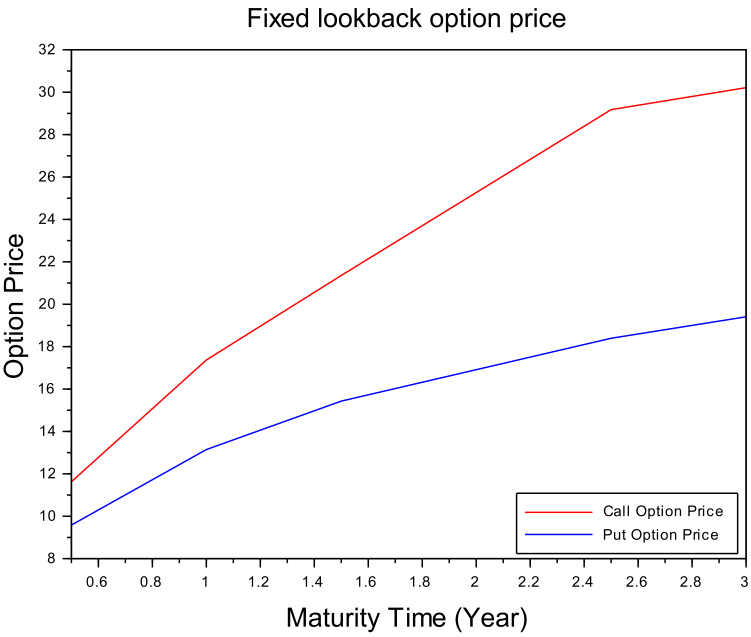

Figure 4. Effect of maturity time on option price Lookback Fixed

4. Conclusion

This study concludes that the Monte Carlo simulation method can approximate the value of Lookback Fixed options with stochastic volatility in the Heston model. The error value decreases as more simulations are performed. The simulation results show that an increase in the strike price leads to a decrease in the value of call options and an increase in the value of put options. Conversely, an increase in the initial stock price will raise the price of call options and lower the price of put options. Additionally, the maturity time parameter also has a positive effect on both call and put options. This model provides investors with valuable, accurate information for options trading, taking into account actual market volatility.

Supplementary Information

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Conflict of interest. The authors declare no conflict of interest.

Data availability. The data supporting this research are available in [24].

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