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Research Article

The Occurrence of a Neimark-Sacker Bifurcation on a Discrete-Time Predator-Prey Model Involving Fear Effect and Linear Harvesting

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Abstract

This study investigates the dynamics of a discrete-time predator-prey model incorporating the fear effect and linear harvesting. The model assumes that both prey and predator populations are harvested proportionally to their densities, while the growth rate of the prey population is negatively impacted by predator presence due to fear. The analytical exploration identifies three fixed points: the extinction point, the predator-free equilibrium, and the coexistence equilibrium. Stability analysis and numerical simulations confirm the occurrence of a Neimark-Sacker bifurcation at the interior equilibrium, demonstrating the model's complex dynamical behaviors. The findings provide insights into population sustainability under different harvesting and predation conditions, highlighting how fear and harvesting jointly influence ecosystem stability.

Keywords : *Discrete-time model · Predator-prey · Fear effect · Harvesting · Neimark-Sacker*
MSC2020 : *92D25 · 92D40 · 37N25 · 39A30 · 39A60*

1. Introduction

In ecosystems, the interaction between predator and prey species is a complex biological phenomenon that has been the focus of research across various disciplines, including ecology and mathematics [1–4]. Mathematical modeling plays a crucial role in understanding predator-prey population dynamics, particularly in describing how ecological factors influence population stability and equilibrium. One factor that has gained increasing attention in recent years is the fear effect, which refers to changes in prey behavior due to the presence of predators, significantly affecting population dynamics [5, 6].

The Lotka-Volterra model [7–9] is one of the classical approaches used in modeling predator-prey interactions. However, this model is often insufficient to describe more complex ecological conditions. As a result, various modifications have been introduced to incorporate additional factors such as the fear effect and harvesting. The fear effect reduces the prey's growth rate, while harvesting, whether applied to prey or predator populations, can impact the overall ecosystem balance. Previous studies have demonstrated that the combination of these two factors can generate highly intricate dynamics, including the occurrence of Neimark-Sacker bifurcation, which signifies qualitative changes in the system [10, 11].

In this study, we develop and analyze a discrete-time predator-prey model that incorporates the fear effect and linear harvesting. This model is based on a modified Lotka-Volterra framework with the

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addition of the fear effect and harvesting parameters applied proportionally to population densities. The primary objective of this research is to explore the system's dynamics, identify feasible equilibrium points, and determine conditions under which the system undergoes Neimark-Sacker bifurcation.

The structure of this article is as follows: In Section 2, we provide the model formulation. Section 3 presents the analytical results of the proposed model, including the existence and stability of equilibrium points. Section 4 provides numerical simulations to confirm the analytical findings and explore the system's dynamic behavior. Finally, Section 5 concludes the study by summarizing the key findings and discussing the ecological implications of the developed model.

2. Model Formulation

We start the model formulation by adapting the Lotka-Volterra equations given by

$$\begin{aligned} \frac{dx}{dt} &= rx - mxy, \\ \frac{dy}{dt} &= nxy - \delta y, \end{aligned} \tag{1}$$

where x and y denote the population density of prey and predator. The parameters r, m, n, and δ respectively express the intrinsic growth rate of prey, predation rate of a predator on prey, conversion of predation rate to the birth rate of a predator, and the natural mortality rate of a predator.

Of the several ecological problems, interactions that occur between two or more populations with predator-prey relationships are rapidly growing. New models are developed to match real-world conditions involving ecological factors. For example, research conducted in [12] investigated the impact of the fear effect on a discrete-time predator-prey relationship with square root functional response. Furthermore, research [13] presented the dynamics of Leslie-Gower discrete models with ratio-dependent response functions, and [14] analyzed the stability of a discrete-time predator prey model with fear and Allee effect.

The Equation (1) becomes a basic model for researchers to develop new models based on the reality that occurs in nature. In this study, we analyzed the influence of two ecological factors, the fear effect [15, 16] and harvesting [17–19]. Assuming that the presence of predators can affect the birth rate of the prey population. Numerous studies have shown that the presence of predators can have an impact on prey, which is more important than direct predation for controlling population dynamics [20]. By applying the fear effect term to model (1), we obtain the following modified Lotka-Volterra model.

$$\begin{aligned} \frac{dx}{dt} &= \frac{rx}{1+ay} - mxy, \\ \frac{dy}{dt} &= nxy - \delta y, \end{aligned} \tag{2}$$

where a is the constant fear level of the prey due to the appearance of the predator.

Another topic that arises in ecological issues is harvesting, which can affect the existence of a population. This involves humans, who need other organisms for food. In general, harvesting is often applied in agriculture, fisheries, and forestry. In mathematical modeling, there are three types of harvesting that can be studied, namely constant harvesting, non-linear harvesting, and proportional harvesting [21]. Harvesting factors can affect the stability of predator-prey models, such as the growth of prey and predator [22, 23]. Assuming the prey and predator populations are harvested proportionally, we get

$$\begin{aligned} \frac{dx}{dt} &= \frac{rx}{1+ay} - mxy - q_1 x, \\ \frac{dy}{dt} &= nxy - \delta y - q_2 y, \end{aligned} \tag{3}$$

where q_1 and q_2, respectively, are the harvesting rates of the prey and predator populations. Furthermore, by assuming that there is competition between individuals in the prey population for food, the model (3) is modified into

$$\begin{aligned} \frac{dx}{dt} &= \frac{rx}{1+ay} - \beta x^2 - mxy - q_1 x, \\ \frac{dy}{dt} &= nxy - \delta y - q_2 y, \end{aligned} \tag{4}$$

where β denotes the rate of population reduction due to competition for food with other individuals in the prey population. In the current development of modeling, first-order models are considered inadequate in describing a more realistic state because the state of the next biological condition depends only on the current state [24]. Therefore, in this study, we used a discrete-time model as the operator. This is because discrete time models have several advantages over continuous models, such as complexity in dynamic behavior and suitability for approximating real data because the data has a discrete formula [10]. By using the forward Euler method as in [], the following discrete model is obtained.

$$\begin{aligned} x_{n+1} &= x_n + h\left[\frac{rx_n}{1+ay_n} - \beta x_n^2 - mx_ny_n - q_1x_n\right], \\ y_{n+1} &= y_n + h\left[nx_ny_n - \delta y_n - q_2y_n\right]. \end{aligned} \tag{5}$$

The parameters used in this study can be seen in the following Table 1.

Table 1. Biological interpretation of parameters

Parameter	Description
a	The fear is constant in the prey population to the predator population.
β	The rate of competition between prey populations.
δ	Natural mortality rate of the predator population.
m	Predator population predation rate on prey.
n	The conversion rate of prey predation.
r	Intrinsic growth rate of the prey population.
q_1	Harvesting rate of the prey population.
q_2	Harvesting rate of the predator population.

3. Analytical Results

In this section, we observe the dynamic behavior of the model (5) analytically, which involves the existence of fixed points and their stability. The fixed point of the model (5) is obtained by solving the following equation.

$$\begin{aligned} x &= x + h\left[\frac{rx}{1+ay} - \beta x^2 - mxy - q_1x\right], \\ y &= y + h\left[nxy - \delta y - q_2y\right]. \end{aligned} \tag{6}$$

The first fixed point is given by the origin $E_0 = (0,0)$, which represents the condition when the prey and predator populations will become extinct. The second fixed point is the axial point $\bar{E} = \left(\frac{r-q_1}{\beta}, 0\right)$, which expresses the condition when the extinction of the predator population occurs. The third fixed point is the interior fixed point given by $E^* = (x^*, y^*)$, where $x^* = \frac{\delta + q_2}{n}$ and y^* is the positive solution to y of the following quadratic equation.

$$p_1y^2 + p_2y + p_3 = 0, \tag{7}$$

where

$$\begin{aligned}
p_1 &= amn,\\
p_2 &= \beta\delta a + \beta a q_2 + mn + a q_1 n,\\
p_3 &= \beta\delta + \beta q_2 + q_1 n - nr.
\end{aligned}$$

This fixed point expresses the condition where the populations of prey and predator maintain their existence in the ecosystem. Using Descartes' rule of signs, the existence of the fixed point E^* is shown by the following Lemma 1.

Lemma 1. *(i) If* $r < \dfrac{(\delta + q_2)\beta + q_1 n}{n}$ *then* $p_3 > 0$*, resulting in the fixed point not existing or no fixed point* E^* *in the interior.*

(ii) If $r > \dfrac{(\delta + q_2)\beta + q_1 n}{n}$ *then* $p_3 < 0$*, so there is one positive root at the fixed point* E^* *in the interior.*

To obtain the behavior of the solution around these fixed points, we apply linearization to model (5), thus obtaining the following Jacobi matrix.

$$J_{(x,y)} = \begin{bmatrix} \zeta_{11} & \zeta_{12} \\ hny & \zeta_{22} \end{bmatrix}, \tag{8}$$

where

$$\begin{aligned}
\zeta_{11} &= 1 + h\left(\frac{r}{1+ay} - 2\beta x - my - q_1\right),\\
\zeta_{12} &= -h\left(\frac{arx}{(1+ay)^2} + mx\right),\\
\zeta_{22} &= 1 + h\left(nx - \delta - q_2\right).
\end{aligned}$$

The stability of each fixed point using Jacobi matrix (8) is given by the following Theorem.

Theorem 2. *Let* $q_1 > r$*. The fixed point of extinction of the entire population* $E_0 = (0,0)$ *is*

(i) sink if $0 < h < 2 \min\left\{\dfrac{1}{q_1 - r}, \dfrac{1}{\delta + q_2}\right\}$*, or;*

(ii) source if $h > 2 \max\left\{\dfrac{1}{q_1 - r}, \dfrac{1}{\delta + q_2}\right\}$*, or;*

(iii) saddle if $\dfrac{2}{q_1 - r} < h < \dfrac{2}{\delta + q_2}$*, or* $\dfrac{2}{\delta + q_2} < h < \dfrac{2}{q_1 - r}$*, or;*

(iv) non-hyperbolic if $h = \dfrac{2}{q_1 - r}$ *or* $\dfrac{2}{\delta + q_2}$*.*

Proof. By calculating the Jacobi matrix (8) at the fixed point $E_0 = (0,0)$, we obtain

$$J_{E_0} = \begin{bmatrix} 1 + h(r - q_1) & 0 \\ 0 & 1 - h(\delta + q_2) \end{bmatrix}, \tag{9}$$

so that two eigenvalues are obtained as follows.

$$\begin{aligned}
\lambda_1 &= 1 + h(r - q_1),\\
\lambda_2 &= 1 - h(\delta + q_2).
\end{aligned}$$

We confirm that

- $|\lambda_1|<1$ when $h<\dfrac{2}{q_1-r}$, $|\lambda_2|<1$ when $h<\dfrac{2}{\delta+q_2}$,
- $|\lambda_1|>1$ when $h>\dfrac{2}{q_1-r}$, $|\lambda_2|>1$ when $h>\dfrac{2}{\delta+q_2}$,
- $|\lambda_1|=1$ when $h=\dfrac{2}{q_1-r}$, $|\lambda_2|=1$ when $h=\dfrac{2}{\delta+q_2}$.

Obeying Lemma 1 in [25], all statements given in Theorem 2 are analytically proven. ■

Theorem 3. *Let* $n<\dfrac{(\delta+q_2)\beta}{(r-q_1)}$ *and* $r>q_1$. *Predator extinction fixed point* $\bar{E}=\left(\dfrac{r-q_1}{\beta},0\right)$ *is*

(i) sink, if $0<h<2\ \min\left\{\dfrac{1}{r-q_1},\dfrac{\beta}{(\delta+q_2)\beta-(r-q_1)n}\right\}$, *or;*

(ii) source, if $h>2\ \max\left\{\dfrac{1}{r-q_1},\dfrac{\beta}{(\delta+q_2)\beta-(r-q_1)n}\right\}$, *or;*

(iii) saddle, if $\dfrac{2}{r-q_1}<h<\dfrac{2\beta}{(\delta+q_2)\beta-(r-q_1)n}$, *or* $\dfrac{2\beta}{(\delta+q_2)\beta-(r-q_1)n}<h<\dfrac{2}{r-q_1}$, *or;*

(iv) non-hyperbolic, if $h=\dfrac{2}{r-q_1}$ *or* $\dfrac{2\beta}{(\delta+q_2)\beta-(r-q_1)n}$.

Proof. By calculating the Jacobi matrix (8) at $\bar{E}=\left(\dfrac{r-q_1}{\beta},0\right)$, we obtain

$$J_{\bar{E}}=\begin{bmatrix}1-h(r-q_1) & -h\left(\dfrac{(ar+m)(r-q_1)}{\beta}\right)\\ 0 & 1+h\left(\dfrac{(r-q_1)n}{\beta}-\delta-q_2\right)\end{bmatrix}, \tag{10}$$

so that the following two eigenvalues are obtained.

$$\lambda_1=1-h(r-q_1),$$
$$\lambda_2=1+h\left(\frac{(r-q_1)n}{\beta}-\delta-q_2\right).$$

We ensure

- $|\lambda_1|<1$ when $h<\dfrac{2}{r-q_1}$, $|\lambda_2|<1$ when $h<\dfrac{2\beta}{(\delta+q_2)\beta-(r-q_1)n}$,
- $|\lambda_1|>1$ when $h>\dfrac{2}{r-q_1}$, $|\lambda_2|>1$ when $h>\dfrac{2\beta}{(\delta+q_2)\beta-(r-q_1)n}$,
- $|\lambda_1|=1$ when $h=\dfrac{2}{r-q_1}$, $|\lambda_2|=1$ when $h=\dfrac{2\beta}{(\delta+q_2)\beta-(r-q_1)n}$.

Following Lemma 1 in [25], all statements given in Theorem 3 are analytically proven. ■

Theorem 4. *Let* $\omega=ny^*\left(\dfrac{arx^*}{(1+ay^*)^2}+mx^*\right)$. *The fixed point of existence of the whole population* $E^*=(x^*,y^*)$ *is*

(i) sink if $h<\dfrac{\beta x^*}{\omega}$ *and* $\beta<\dfrac{h^2\omega+4}{2hx^*}$, *or;*

(ii) source if $h>\dfrac{\beta x^*}{\omega}$ *and* $\beta<\dfrac{h^2\omega+4}{2hx^*}$, *or;*

(iii) saddle if $\beta>\dfrac{h^2\omega+4}{2hx^*}$, *or;*

(iv) non-hyperbolic if $h\neq\dfrac{2}{\beta x^*}$ *and* $\beta=\dfrac{h^2\omega+4}{2hx^*}$, *or;*

(iv) *complex if* $h = \dfrac{\beta x^*}{\omega}$ *and* $\omega > \dfrac{\beta^2 (x^*)^2}{4}$.

Proof. By calculating the Jacobi matrix (8) around $E^* = (x^*, y^*)$, we obtain

$$J_{E^*} = \begin{bmatrix} 1 - h\beta x^* & -h\left(\dfrac{arx^*}{(1+ay^*)^2} + mx^*\right) \\ hny^* & 1 \end{bmatrix}, \tag{11}$$

By solving the matrix (11) we get $Tr = 2 - h\beta x^*$ and $Det = 1 - h\beta x^* + h^2 ny^* \left(\dfrac{arx^*}{(1+ay^*)^2} + mx^*\right)$. Let $\omega = ny^* \left(\dfrac{arx^*}{(1+ay^*)^2} + mx^*\right)$. Then, since $1 - h\beta x^* = Tr - 1$ then for Det it can be rewritten into

$$Det = Tr - 1 + h^2 \omega.$$

When $F(1) = h^2\omega$, then using Lemma 2 in [25], we can see the biological behavior at the fixed point E^*. Thus

- $Det < 1$ when $h < \dfrac{\beta x^*}{\omega}$, and $F(-1) > 0$ when $\beta < \dfrac{h^2\omega + 4}{2hx^*}$,
- $Det > 1$ when $h > \dfrac{\beta x^*}{\omega}$, and $F(-1) > 0$ when $\beta < \dfrac{h^2\omega + 4}{2hx^*}$,
- $F(-1) < 0$ when $\beta > \dfrac{h^2\omega + 4}{2hx^*}$,
- $Tr \neq 0.2$ when $h \neq \dfrac{2}{\beta x^*}$, and $F(-1) = 0$ when $\beta = \dfrac{h^2\omega + 4}{2hx^*}$,
- $Det = 1$ when $h = \dfrac{\beta x^*}{\omega}$, and $Tr^2 - 4Det < 0$ when $\omega > \dfrac{\beta^2 (x^*)^2}{4}$.

Obeying Lemma 2 in [25], all statements given in Theorem 4 are analytically proven. ■

4. Numerical Results

In this section, we will numerically demonstrate the dynamic behavior of the model (5) based on the analytical results in the previous section. We start the simulation by selecting the parameter values as follows.

$$r = 1.4,\ a = 0.09,\ \beta = 0.07,\ \delta = 0.09,\ q_1 = 1.6,\ q_2 = 0.7,\ m = 0.3,\ n = 0.2,\ h = 0.15. \tag{12}$$

We obtain the phase portrait and *time-series* on Figure 1. Four initial values are given around the origin. All solutions provided by these initial values converge to the fixed point $E_0 = (0,0)$ when the harvesting rate of the prey population is greater than the intrinsic growth rate. In Figure 1 it can be seen that the solution moves towards the fixed point E_0 which means that taking these parameter values, the solution will lead to the extinction of the entire population of both prey and predator.

Further simulations are performed by lowering the parameter value q_1 to 1.3. Based on the existence and stability conditions of the fixed point $\bar{E}$, it can be confirmed that the previously stable $E_0 = (0,0)$ becomes unstable, accompanied by the emergence of the fixed point $\bar{E} = (1.43, 0)$ which sinks. To demonstrate this condition, some initial conditions around $\bar{E}$ are chosen and then simulated to obtain Figure 2. Based on Figure 2, it can be seen that when the harvesting of the prey population is reduced to 1.3, all solutions move towards to the fixed point $\bar{E}$ which is the point of existence of the prey population and the extinction of the predator population.

To show the stability at the fixed point E^*, the parameters are chosen as in the previous simulation, in this case by lowering the parameter value q_1 to 0.5. As a result, E^* becomes a sink. By selecting some initial conditions, this condition is simulated in Figure 3 in the form of phase portrait

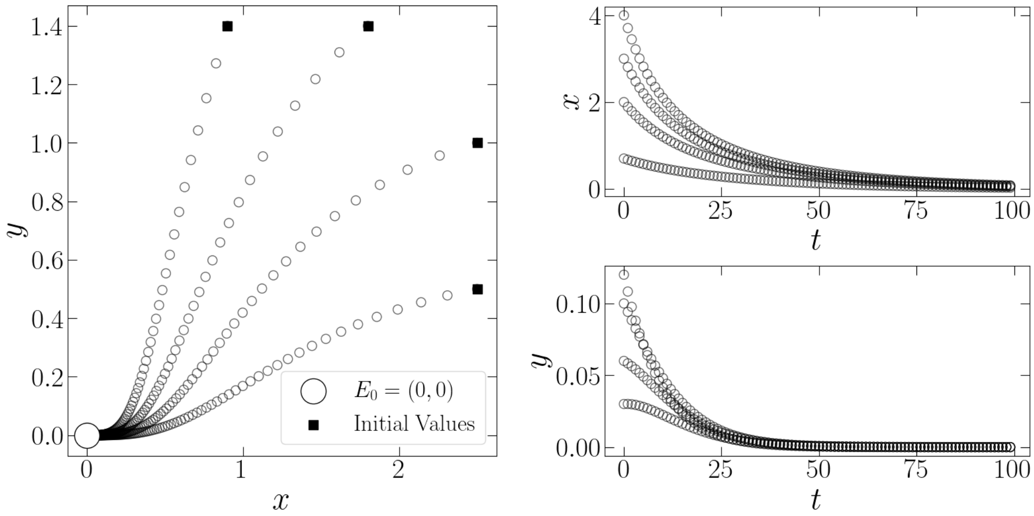

Figure 1. Phase portrait and time-series to see the effect of harvesting rate on the dynamics of the model (5) using the parameter value $q_1 = 1.6$

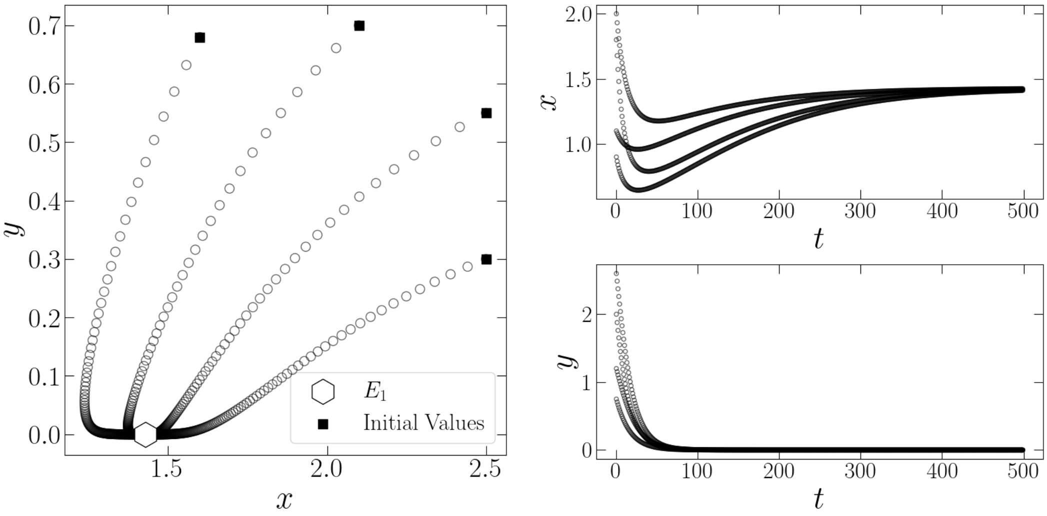

Figure 2. Phase portrait and time-series to see the effect of harvesting rate on the dynamics of the model (5) using the parameter value $q_1 = 1.3$

and time series. It is shown that by lowering the value of the population harvesting rate parameter on prey, the prey and predator populations will still exist and not go extinct.

Finally, similar parameter values were used in the previous simulation, and the step-size value (h) was changed to 0.7. By choosing some initial conditions around it, these conditions are simulated on Figure 4 in the form of phase portraits. Numerically, Figure 4 shows the occurrence of Neymark-Sacker bifurcation at the unique interior point E^*. The previous asymptotically stable (sink) E^* loses its stability accompanied by the emergence of an interesting phenomenon, namely limit cycle. This means that for each initial value, the solution is still in the same orbit and isolates the interior point of E^* so that the population can be maintained periodically. Biologically, it is shown that the initial conditions of the densities of the two populations are close enough to the fixed interior point to preserve their existence.

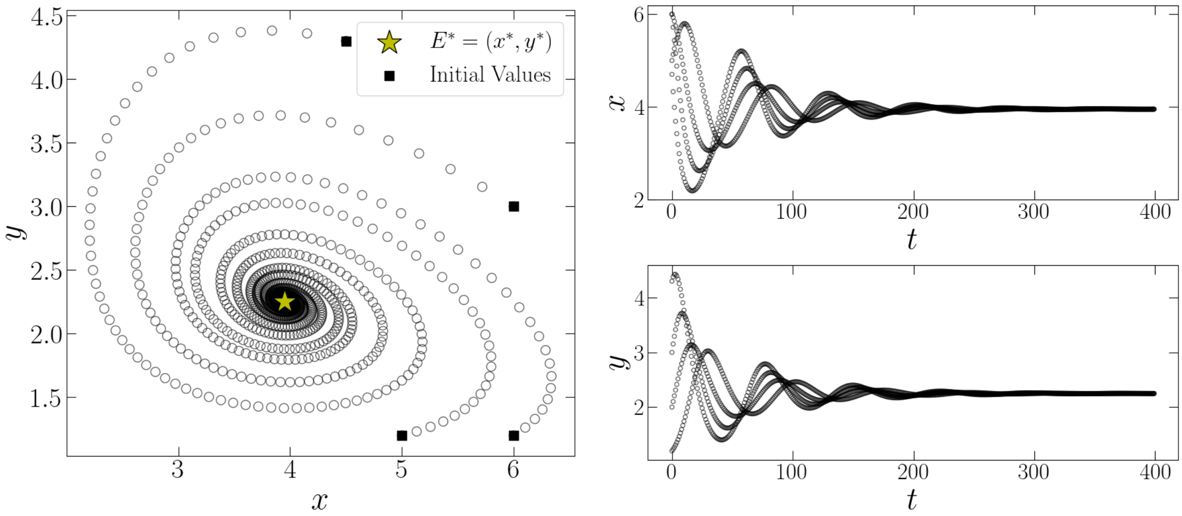

Figure 3. Phase portrait and time-series to see the effect of harvesting rate on the dynamics of the model (5) using the parameter value $q_1 = 0.5$

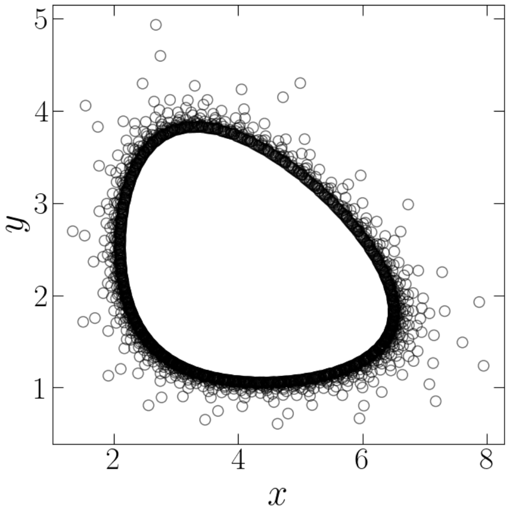

Figure 4. Phase portrait to see the effect of changing the value of h on the dynamics of the model (5) by using the parameter value $q_1 = 0.5$

5. Conclusion

The discrete-time predator-prey model, which incorporated the fear effect and linear harvesting has been discussed in this paper. The step size parameter h plays crucial role in the dynamical behaviors of the model. Numerical simulations show when the prey harvesting rate exceed the intrinsic growth rate, both populations become extinct. However, as the harvesting reduced, the predator-free equilibrium becomes stable, and further reduction on the harvesting rate parameter value in the prey population leads to the coexistence of both populations. Further, we also verified the occurrence of Neimark-Sacker bifurcation as we varied the step size h. These results could emphasized the importance of harvesting management in the predator-prey interactions in order to maintain the ecological balance.

Supplementary Information

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Conflict of interest. The authors declare no conflict of interest.

Data availability. All data in this paper are hypothetically provided.

References

[1] Anggriani N, Panigoro HS, Rahmi E, Peter OJ, Jose SA. A predator–prey model with additive Allee effect and intraspecific competition on predator involving Atangana–Baleanu–Caputo derivative. Results in Physics. 2023;49(April):106489. [Crossref].
[2] Panigoro HS, Rahmi E, Peter OJ. Recognizing the complexity of a predator-prey relationship with allee and fear effects in a discrete-time model. vol. 050003; 2024. p. 050003. [Crossref].
[3] Bassim WB, Salem AJ, Ali AH. A new analytical study of prey–predator dynamical systems involving the effects of Hide-and-Escape and predation skill augmentation. Results in Control and Optimization. 2024;16:100449. [Crossref].
[4] Panigoro HS, Rahmi E, Bonyah E, Akgül A, Jose SA. Dynamical Behaviors of a Fractional-Order Predator–Prey Model: Insights into Multiple Predators Competing for a Single Prey. Fractals. 2025;2540077:1-18. [Crossref].
[5] Xue Y. Impact of both-density-dependent fear effect in a Leslie–Gower predator–prey model with Beddington–DeAngelis functional response. Chaos, Solitons & Fractals. 2024;185(May):115055. [Crossref].
[6] Zhang C. The effect of the fear factor on the dynamics of an eco-epidemiological system with standard incidence rate. Infectious Disease Modelling. 2024;9(1):128-41. [Crossref].
[7] Lotka AJ. Elements of Physical Biology. Nature. 1925;116(2917):461-1. [Crossref].
[8] Volterra V. Variazioni e fluttuazioni del numero d'individui in specie animali conviventi. Mem Acad Lincei Roma. 1926;2:31-113. [Website].
[9] Berryman AA. The Orgins and Evolution of Predator-Prey Theory. Ecology. 1992;73(5):1530-5. [Crossref].
[10] Panigoro HS, Resmawan R, Rahmi E, Beta MA, Sidik ATR. The Existence of a Limit-Cycle of a Discrete-Time Lotka-Volterra Model with Fear Effect and Linear Harvesting. E3S Web of Conferences. 2023;400:03003. [Crossref].
[11] Shang Z, Qiao Y. Complex dynamics of a four-species food web model with nonlinear top predator harvesting and fear effect. Mathematics and Computers in Simulation. 2024;223(March):458-84. [Crossref].
[12] Santra PK. Fear effect in discrete prey-predator model incorporating square root functional response. Jambura Journal of Biomathematics. 2021;2(2):51-7. [Crossref].
[13] Mokodompit R, Nurwan N, Rahmi E. Bifurkasi Periode Ganda dan Neimark-Sacker pada Model Diskret Leslie-Gower dengan Fungsi Respon Ratio-Dependent. Limits: Journal of Mathematics and Its Applications. 2020;17(1):19. [Crossref].
[14] Panigoro HS, Rahmi E, Peter OJ. Recognizing the complexity of a predator-prey relationship with allee and fear effects in a discrete-time model. vol. 050003; 2024. p. 050003. [Crossref].
[15] Barman D, Roy J, Alrabaiah H, Panja P, Mondal SP, Alam S. Impact of predator incited fear and prey refuge in a fractional order prey predator model. Chaos, Solitons & Fractals. 2021;142:110420. [Crossref].
[16] Panigoro HS, Rahmi E. Impact of Fear and Strong Allee Effects on the Dynamics of a Fractional-Order Rosenzweig-MacArthur Model. In: Banerjee S, Saha A, editors. Nonlinear Dynamics and Applications. Springer Proceedings in Complexity. Cham: Springer International Publishing; 2022. p. 611-9. [Crossref].
[17] Maisaroh S, Resmawan R, Rahmi E. Analisis Kestabilan Model Predator-Prey dengan Infeksi Penyakit pada Prey dan Pemanenan Proporsional pada Predator. Jambura Journal of Biomathematics. 2020;1(1):8-15. [Crossref].
[18] Nisar KS, Kumar GR, Ramesh K. The study on the complex nature of a predator-prey model with fractional-order derivatives incorporating refuge and nonlinear prey harvesting. AIMS Mathematics. 2024;9(5):13492-507. [Crossref].
[19] Wang SxWZc, Ruan S. Hopf bifurcation in an age-structured predator – prey system with Beddington – DeAngelis functional response and constant harvesting. Journal of Mathematical Biology. 2024;88(5):1-74. [Crossref].
[20] Gotceitas V, Brown JA. Substrate selection by juvenile Atlantic cod (Gadus morhua): effects of predation risk. Oecologia. 1993;93(1):31-7. [Crossref].

[21] Purnomo AS, Darti I, Suryanto A. Dynamics of eco-epidemiological model with harvesting. In: AIP Conference Proceedings. vol. 1913; 2017. p. 020018. [Crossref].

[22] Rowe S, Hutchings JA, Bekkevold D, Rakitin A. Depensation, probability of fertilization, and the mating system of Atlantic cod (Gadus morhua L.). ICES Journal of Marine Science. 2004;61(7):1144-50. [Crossref].

[23] Baum JK, Myers RA, Kehler DG, Worm B, Harley SJ, Doherty PA. Collapse and Conservation of Shark Populations in the Northwest Atlantic. Science. 2003;299(5605):389-92. [Crossref].

[24] Panigoro HS, Rahmi E, Suryanto A, Darti I. A fractional order predator–prey model with strong allee effect and Michaelis–Menten type of predator harvesting. AIP Conference Proceedings. 2022;2498(1):20018. [Crossref].

[25] Panigoro HS, Rahmi E, Achmad N, Mahmud SL, Resmawan R, Nuha AR. A discrete-time fractional-order Rosenzweig-Macarthur predator-prey model involving prey refuge. Communications in Mathematical Biology and Neuroscience. 2021;2021:Article ID 77. [Crossref].