Indonesian Journal of Computational and Applied Mathematics
https://ejournal.gammarisepub.com/index.php/indocam
Volume 1, Issue 2, pages 89–95, June 2025
https://doi.org/10.64182/indocam.v1i2.33, E-ISSN 3090-5281

Research Article

Analysis of a Predator-Prey Model incorporating Prey Cannibalism and Intraspecific Competition on Predator

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Abstract

In this research, we formulated a predator-prey model by considering cannibalism in the prey and intraspecific competition on predator population. We found three types of equilibrium points existed under certain condition, except the extinction of all population equilibrium point. Further, we analyzed the local stability of each equilibrium point via linearization method. We found that the extinction of all population equilibrium point is always unstable and the other points locally asymptotically stable under some conditions. Finally, the numerical simulation carried out to verify the analytical results and to perform the impact of prey cannibalism rate.

Keywords : *Prey cannibalism · Local stability · Predator-prey model*
MSC2020 : *34D20 · 92D25 · 00A69 · 37N25*

1. Introduction

Predator–prey modeling has evolved as an essential tool in understanding the intricate interactions that govern ecological systems. Early models, such as the Lotka–Volterra equations, laid the foundation for describing population fluctuations between predators and their prey. Over time, these models have been refined to incorporate various biological and environmental complexities, including carrying capacity, time delays, stage structures, and behavioral adaptations. Such advancements have improved the predictive capabilities of models and enhanced their applicability to conservation management and ecosystem restoration. Recent developments emphasize integrating external factors such as disease transmission, habitat heterogeneity, and behavioral responses, which significantly influence population stability and resilience [1–5]. This growing body of research underscores the importance of extending classical models to better reflect ecological realities and to provide more accurate insights for biodiversity management and ecological forecasting.

Cannibalism, defined as the act of consuming individuals of the same species, is a widespread phenomenon observed across various taxa in both terrestrial and aquatic ecosystems. It manifests in numerous forms, such as egg cannibalism in the predatory bug Arma custos [6], intraspecific predation in the invasive zooplanktivore Bythotrephes cederströmii [7], and sexual cannibalism in certain arachnids and mantids [8]. In aquatic systems, cannibalism occurs in fish and zooplankton, influencing early life stage survival [9], while in amphibians and reptiles, larval cannibalism can reduce competition under nutrient-limited conditions [10]. In mammals, cannibalistic behavior may emerge under environmental stressors, including food scarcity [8]. The ecological impacts of cannibalism are multifaceted—it can regulate population density, alter demographic structures, and modify community interactions. The occurrence of cannibalism is often linked to life stage, resource availability,

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received: 18 July 2025, revised: 7 October 2025, accepted: 7 October 2025, published Online: 7 October 2025

GAMMARISE PUBLISHING

and environmental pressures, making it a critical factor in shaping population dynamics and species survival strategies [6–10].

In recent years, predator–prey models have been expanded to explicitly incorporate cannibalistic interactions, especially among prey populations, to better capture their complex ecological consequences. Mathematical frameworks such as the Leslie–Gower model with prey cannibalism demonstrate that cannibalism can alter equilibrium stability, potentially transforming a stable system into one exhibiting periodic oscillations or limit cycles [1, 11]. Other studies indicate that prey cannibalism may reduce predator density while affecting prey population persistence [12]. Furthermore, the inclusion of ecological factor such as intraspecific competition within predator population could present a novel extension to these dynamics. Intraspecific competition between predators for prey occurs when the predator to prey ratio is large enough, causing individuals in the predator population to experience decreased fitness due to lack of food [13].

Based on the above discussion, in Section 2 the predator-prey model incorporating cannibalism on prey and intraspecific competition among predator is developed. In Section 3, we analyze the local stability each of the equilibrium points. We also carried out some numerical simulations to verify the analytical results. Finally, the conclusions are given in Section 4.

2. Mathematical Model

In this model construction, we modified the basic Lotka-Volterra model with the presence of cannibalism in prey population in the last two terms in the first equation of system (1). The term $\dfrac{cN^2}{\alpha+N}$ denotes the cannibalism in prey which follows the Holling type II predation term and c_1N represent the conversion of cannibalism into prey birth, respectively. Taking account the intraspecific competition in the predator population, the model becomes

$$\begin{aligned}\frac{dN}{dt} &= rN - a_1NP + bN - \frac{cN^2}{\alpha+N},\\ \frac{dP}{dt} &= a_2NP - mP^2 - dP,\end{aligned} \tag{1}$$

where $b < c$, $N \geq 0$ and $P \geq 0$ represent the densities of prey and predator at time t, respectively. The description of all parameters in system (1) are presented in Table 1.

Table 1. Parameter Model

Parameter	Biological interpretation
r	Intrinsic grow rate of prey population
a_1	Rate of decline of prey population due interaction with predator
b	Conversion rate of cannibalism into prey birth
c	Rate of cannibalism in prey
α	The half-saturation constant of cannibalism
a_2	Conversion rate of prey biomass into predator birth
m	Death rate due to competition between predator
d	Natural death rate of the predator population

3. Analytical Results

The equilibrium point was obtained as follows $\dfrac{dN}{dt} = \dfrac{dP}{dt} = 0$.

$$\begin{aligned} rN - a_1NP + bN - \frac{cN^2}{\alpha+N} &= 0,\\ a_2NP - mP^2 - dP &= 0.\end{aligned} \tag{2}$$

The equilibrium points are obtained as follows.

1. The extinction of all populations point $E_0 = (0,0)$
2. The extinction of the prey population point $E_P = \left(0, -\frac{d}{m}\right)$ which is not defined.
3. The extinction of the predator population point $E_N = \left(\frac{(r+b)\alpha}{c-(r+b)}, 0\right)$, exist when $c > r+b$.
4. The existence of all populations point $E^* = (N^*, P^*)$ with $P^* = \frac{a_2N^* - d}{m}$ and N^* satisfies the quadratic equation

   $$(a_1a_2)N^2 + (cm + a_1a_2\alpha - a_1d - (r+b)m)N - \alpha((r+b)m + a_1d) = 0, \tag{3}$$

   with the discriminant value $\Delta = (cm + a_1a_2\alpha - a_1d - (r+b)m)^2 + 4a_1a_2\alpha((r+b)m + a_1d)$ is always a positive value. According to the Descarte's rule of signs, equation (3) can only have one positive real root since there is only one sign change.

3.1. Local Stability

The Jacobian matrix for the system (1) is given by

$$J_{(\hat{N},\hat{P})} = \begin{bmatrix} r - a_1\hat{P} + b - \dfrac{c\hat{N}(2\alpha + \hat{N})}{(\alpha + \hat{N})^2} & -a_1\hat{N} \\ a_2\hat{P} & a_2\hat{N} - 2m\hat{P} - d \end{bmatrix}. \tag{4}$$

By calculating the Jacobian matrix around the equilibrium point, the local stability at each equilibrium point is determined as follows.

1. Local stability of equilibrium point E_0
   By substituting $E_0 = (0,0)$ into the Jacobian matrix (4), we obtain

   $$J_{(E_0)} = \begin{bmatrix} r+b & 0 \\ 0 & -d \end{bmatrix}.$$

   The stability of the equilibrium point E_0 is explained in the following theorem

   Theorem 1. *The equilibrium point $E_0 = (0,0)$ is always unstable.*

   Proof. By solving $det(J_{E_0} - \lambda I) = 0$, we obtain

   $$\begin{aligned} \lambda_1 &= r+b \quad \text{dan} \\ \lambda_2 &= -d_2. \end{aligned}$$

   It can be seen that the eigen value λ_1 is always positive. Therefore, the equilibrium point $E_0 = (0,0)$ is always unstable. ∎

2. Local stability of equilibrium point E_N
   The Jacobian matrix (4) for $E_N = \left(\frac{(r+b)\alpha}{c-(r+b)}, 0\right)$ is

   $$J|_{E_P} = \begin{bmatrix} \dfrac{(r+b)((r+b)-c)}{c} & \dfrac{a_1(r+b)\alpha}{(r+b)-c} \\ 0 & \dfrac{d(c-(r+b)) - a_2(r+b)\alpha}{(r+b)-c} \end{bmatrix}.$$

   Then the stability of the equilibrium of point E_N is explained in the following theorem.

Theorem 2. *The equilibrium of point* $E_N = \left(\frac{\alpha(d_1-b)}{c-b+d_1}, 0\right)$ *is locally asymptotically stable if* $d > \frac{a_2(r+b)\alpha}{(c-(r+b))}$.

Proof. By solving $det(J_{E_2} - \lambda I) = 0$, we obtain

$$\lambda_1 = \frac{(r+b)((r+b)-c)}{c},$$
$$\lambda_2 = \frac{d(c-(r+b)) - a_2(r+b)\alpha}{(r+b)-c}.$$

From the existence condition of E_N, then $\lambda_1 < 0$. Therefore, E_N is locally asymptotically stable if $d > \frac{a_2(r+b)\alpha}{(c-(r+b))}$. ■

3. Local stability of equilibrium point E^*
   By substituting $E^* = (N^*, P^*)$ to Jacobian matrix (4) we have

$$J|_{E^*=(N^*,P^*)} = \begin{bmatrix} r - a_1P^* + b - \frac{cN^*(2\alpha+N^*)}{(\alpha+N^*)^2} & -a_1N^* \\ a_2P^* & a_2N^* - 2mP^* - d \end{bmatrix}. \tag{5}$$

Since $P^* = \frac{a_2N^* - d}{m}$, the Jacobian (5) can be written as

$$J|_{E^*} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix},$$

where

$$J_{11} = r - \frac{a_1(a_2N^*-d)}{m} + b - \frac{2cN^*}{\alpha+N^*} + \frac{cN^{*2}}{(\alpha+N^*)^2},$$
$$J_{12} = -a_1N^*,$$
$$J_{21} = \frac{(a_2N^*-d)a_2}{m},$$
$$J_{22} = d - a_2N^*.$$

Then the stability of the E_3 equilibrium point is explained in the following theorem.

Theorem 3. *The equilibrium point* $E_3 = (N^*, P^*)$ *is locally asymptotically stable if* $r + b < \frac{cN^{*2}}{(\alpha+N^*)^2} - \frac{2cN^*}{\alpha+N^*} - \frac{a_1(a_2N^*-d)}{m}$.

Proof. The characteristic equation of Jacobian matrix eq. (5) is

$$\lambda^2 - trace(J)\lambda + \det(J) = 0, \tag{6}$$

where the determinant and trace are, respectively given by

$$\begin{aligned} \det(J) &= J_{11}J_{22} - J_{12}J_{21} \\ &= \left(r + b + \frac{cN^{*2}}{(\alpha+N^*)^2} - \frac{2cN^*}{\alpha+N^*} - \frac{a_1(a_2N^*-d)}{m}\right)(d - a_2N^*) \end{aligned}$$

$$+a_1 a_2 N^* \frac{(a_2 N^* - d)}{m}$$

and

$$\begin{aligned} trace(J) &= J_{11} + J_{22} \\ &= \left(r + b + \frac{cN^{*2}}{(\alpha + N^*)^2} - \frac{2cN^*}{\alpha + N^*} - \frac{a_1(a_2N^* - d)}{m} \right) + d - a_2 N^*. \end{aligned}$$

Using the Routh-Hurwitz criteria, the equilibrium point E^* is locally asymptotically stable if satisfies $trace(J) < 0$ and $\det(J) > 0$. Since $a_2N^* - d > 0$, then if $r + b < \frac{cN^{*2}}{(\alpha + N^*)^2} - \frac{2cN^*}{\alpha + N^*} - \frac{a_1(a_2N^* - d)}{m}$, we have $trace(J) < 0$ and $\det(J) > 0$. Therefore, E^* is locally asymptotically stable. ■

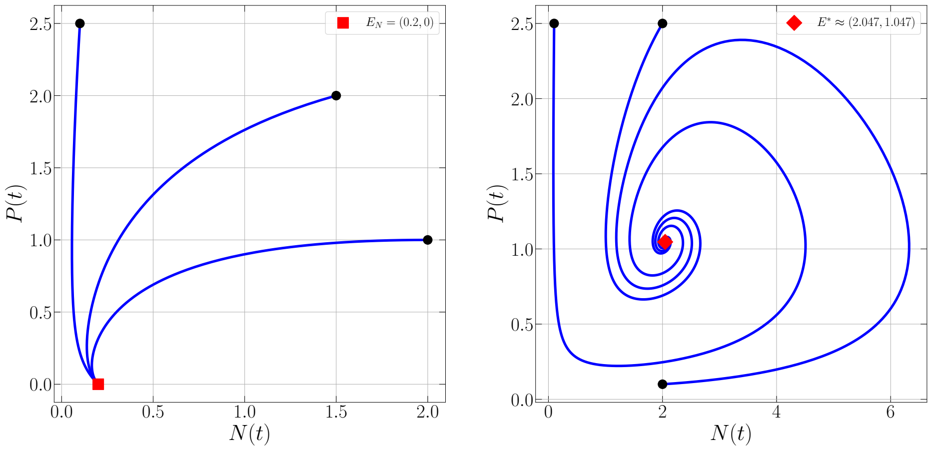
3.2. Numerical Simulation

In this section, the dynamics of the model (1) are discussed numerically using Python software and the 4th order Runge-Kutta method. Based on the previous results, there is one undefined point and three points that are locally stable by considering the stability conditions. This numerical simulation was carried out to confirm the analytical results and to show the impact of prey cannibalism rate to the system. Using hypothetical parameter values, as in Table 2, we perform the following numerical simulations.

Table 2. Parameter Values

Simulations	r	a_1	a_2	b	c	α	d	m
Simulation 1	0.3	0.2	0.1	0.1	0.6	0.1	0.1	0.1
Simulation 2	0.3	0.2	0.1	0.1	0.2	0.1	0.1	0.1
Simulation 3	0.3	0.2	0.1	0.1	-	0.1	0.1	0.1

The first simulation begins by selecting parameter values in Simulation 1 in Table 2. Based on the values, we have an unstable equilibrium point $E_0 = (0,0)$ and a locally asymptotically stable equilibrium point $E_N = (0.2, 0)$ since $0.1 = d > \frac{a_2(r+b)\alpha}{(c-(r+b))} = 0.02$. A phase portrait in Figure 1(a) show that all solutions tend to $E_N = (0.2, 0)$. Thus, it proves that Theorem 1 and Theorem 2 are satisfied.



(a) Phase portrait around E_N (b) Phase portrait around E^*

Figure 1. Phase portrait of model (1) using parameter values as in Table 2

If we decrease the value of cannibalism rate to 0.2 and other parameter values are fixed as in Simulation 2 in Table 2, we have an interior equilibrium point $E^* = (2.047, 1.047)$ with the eigen values $\lambda_{1,2} = -0.0568 \pm 0.2014I$. With three initial values, all solutions converge to $E^* = (2.047, 1.047)$ as shown in Figure 1(b).

The latest simulation is given to show numerically the impact of prey cannibalism rate c using parameter values Simulation 3 in Table 2 and the rate of cannibalism in prey varied with $c \in [0.1, 0.2, 0.3, 0.4, 0.5, 0.6]$. In Figure 2, we observe the dynamical behavior of model (1) when c increases. It is shown that as c increases, both prey and predator population decreases.

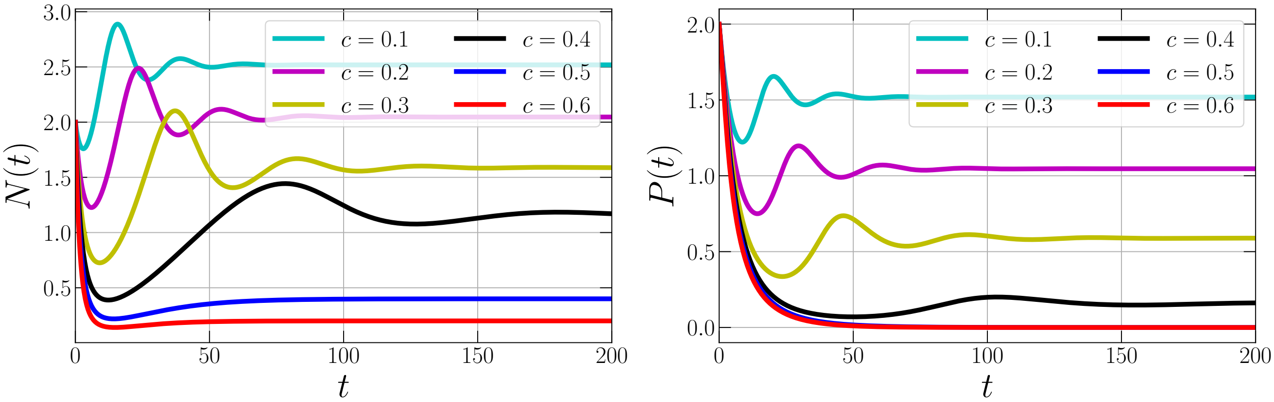


Figure 2. The impact of cannibalism rate in prey with different values of c

4. Conclusion

This research investigated the dynamics of the predator-prey model with cannibalism in prey and intraspecific competition on predator. The model has three equilibrium points where the extinction of all populations point always unstable and the other equilibrium points, i.e. the extinction of the predator population point and the existence of all populations point are conditionally asymptotically stable. We also observe the impact of prey cannibalism rate to the dynamics behavior of model (1). For future work, ones can analyze the global stability of the model.

Supplementary Information

Acknowledgements. We're very thankful to the editors and reviewers for their helpful notes and all the work they put into making this better.

Funding. This research received no external funding.

Author Contributions. **Meiske Biduli:** Conceptualization, Writing -Review & Editing, Supervision, Project Administration. **Emli Rahmi:** Conceptualization, Methodology, Validation, Writing -Review & Editing, Project Administration, Funding Acquisition. **Salmun K. Nasib:** Supervision, Methodology, Validation, Resources. All authors read and approved the final manuscript.

Conflict of interest. The authors declare no conflict of interest.

Data availability. All data presented are hypothetical and cited from prior publications.

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