Indonesian Journal of Computational and Applied Mathematics
https://ejournal.gammarisepub.com/index.php/indocam
Volume 1, Issue 2, pages 63–75, June 2025
https://doi.org/10.64182/indocam.v1i2.32, E-ISSN 3090-5281

Research Article

Dynamic Analysis of the Modified Leslie Gower Model with Harvesting of Prey and Holling Type II Functional Response

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Abstract

This article studies the Modified Leslie-Gower model with constant business harvesting on the prey and functional response of Holling Type II. This approach is more realistic in several phenomena, one of which is in the phenomenon of rice fields, jali snakes, and owls. The research method begins by determining the assumptions for constructing models, stability analysis, and numerical simulations. Analysis of the equilibrium points was carried out to determine the condition of its stability locally using the Jacobian approach and the Routh-Hurwitz criteria by obtaining five existing points. Analysis of stability using the Jacobian matrix shows that the equilibrium point is an asymptotic node in certain conditions. Numerical simulations are carried out to determine the suitability of the results of the analysis using the software Maple and Python. Numerical simulation results show differences in the value of environmental carrying capacity affect changes in system solutions. Thus, the change in the value of carrying capacity does not always produce the same stability because the stability of the system depends on the sensitivity of the parameter to the overall dynamic structure. This finding provides a foundation for the management of biological resources, especially in controlling harvesting so that the population remains balanced.

Keywords : *Modified Leslie-Gower · Holling Type II · Harvesting*
MSC2020 : *34D20 · 92D25 · 00A69 · 37N25*

1. Introduction

Ecology is the science that studies interactions between living things and their environment. These interactions form an ecosystem, which is an ecological system based on the reciprocal relationship between organisms and their environment [1]. Mathematical models in ecology help explain population interaction issues. One such model is the predator-prey model, introduced by Alfred Lotka in 1925 and Vito Volterra in 1926. This model is known as the Lotka-Volterra model [2]. The interplay between predator and prey species in ecosystems is a complicated biological phenomenon that has garnered academic attention across multiple disciplines, including ecology and mathematics [3–5].

According to the Lotka-Volterra model, a prey population grows exponentially in the absence of a predator population. Thus, the prey population will continue to increase indefinitely. In 1948, Leslie-Gower modified the Lotka-Volterra predator-prey model, with predator population growth following a logistic model. The environmental carrying capacity parameter of the predator population was changed to be proportional to the number of available prey. This model is called the Leslie-Gower predator-prey model [6]. The main element of the predator-prey interaction model is the functional response, which depends solely on the density of the prey population [7]. Holling introduced three types of functional responses: Type I, II, and III [8].

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received: 16 July 2025, revised: 14 August 2025, accepted: 4 September 2025, published Online: 25 September 2025

The dynamics of predators and prey are influenced by the effects of harvesting, a tool used to control the overgrowth of certain organisms. There are three possible harvesting approaches: constant yield harvesting, constant effort harvesting, and Michaelis-Menten type harvesting [9]. One approach used in this study is constant effort harvesting. This approach is considered more realistic in some cases that depend on actual catches, such as in the fisheries sector, poaching, or pest control based on population detection. One real-world example in environmental management is the population growth of rice field rats (*Rattus argentiventer*), known as one of the main pests of rice plants. Uncontrolled rice field rat attacks can cause significant economic and ecological losses, as rats have a very high reproductive capacity and cause extensive damage to crops. Consequently, additional strategies are needed to control rat populations, one of which is through a harvesting approach.

In the natural food chain, rats have natural enemies: the jali snake (*Ptyas korros*) and the owl (*Tyto alba*). *Ptyas korros* is used as a predator against rats that damage crops and harm farmers [10]. Owls are very effective in controlling field rats because they require low costs, little energy, cause no environmental damage, and are easy to implement in communities [11]. Rats and snakes are usually modeled using the Holling type II functional response, which includes the predation saturation factor. In contrast, owls as super predators are modeled using a modified Leslie–Gower approach. This approach does not explicitly include saturation factors in the Holling functional response but considers them implicitly through carrying capacity, which depends on predator numbers. Thus, owl population growth slows and approaches zero as it reaches its maximum capacity, which depends on predator numbers, without factoring in capture or handling time as in Holling type II. The interaction of rats, snakes, and owls involves three species in an ecosystem. These interactions can be formulated into a predator-prey model, as in Listyana et al. [12] in their study, "Bifurcation Analysis in a Mathematical Model of Predator-Prey with Two Predators."

Several previous researchers have discussed the Modified Leslie-Gower three-species model with harvesting on prey, as conducted by Suryanto et al. [13], which is related to the modified logistic model by adding two important components: harvesting on prey and the presence of feedback control. Other studies have also discussed the population dynamics of predator-prey using the Modified Leslie-Gower model and the Holling type II functional response conducted by Aziz-Alaoui and Okiye [14] that shows the solution of this model is limited *(bounded)* where the population will not grow infinitely and shows global stability for the interior equilibrium point. The author uses the Holling type II functional response because the jali snake, as a predator, has a handling time, so it cannot prey indefinitely. The Holling type II functional response has been studied in research conducted by Liu and Chen [15]. This research demonstrates that periodic disturbances in nature can cause significant changes in animal population dynamics, resulting in complex and chaotic behavior.

Based on this background, the authors are interested in mathematically examining predator-prey interactions among three species in rice fields: the rice rat (*Rattus argentiventer*), the barn snake (*Ptyas korros*), and the barn owl (*Tyto alba*). This study uses the modified Leslie-Gower approach and the Holling Type II functional response. The model also accounts for harvesting of the prey population. The research began with a literature review to define the phenomenon under investigation. It then involved constructing the model, determining the equilibrium point, analyzing stability, conducting numerical simulations, and drawing conclusions. The research is entitled "*Dynamic Analysis of the Modified Leslie-Gower Model with Harvesting of Prey and Holling Type II Functional Response*".

2. Model Formulation

2.1. Predator-prey model

The modified Leslie-Gower model with two species is a development of research by Aziz-Alaoui and Okiye [14] that examines ecosystem dynamics in a model that has been modified from the Leslie-

Gower model given by

$$\begin{aligned}\frac{dx}{dt} &= rx - bx^2 - \frac{\alpha xy}{x+K},\\ \frac{dy}{dt} &= sy\left(1 - \frac{\beta y}{x+e}\right).\end{aligned} \tag{1}$$

The variables x and y represent the population densities of the prey species and the predator species respectively at time t. The variables in this model include the growth of prey (r), the parameter of the extent to which the environment provides protection for prey (K), the capture coefficient (q), the harvesting effort (H), predation on prey (α), the growth rate of predator (s), the maximum value of the reduction in the predator population due to competition between individuals for prey (β), and the contribution of the environment in providing protection for predator (e).

The predator-prey model with a Holling type II functional response has been studied by Liu and Chen [15] as follows.

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1 - \frac{bx}{K}\right) - \frac{\alpha xy}{ax+1},\\ \frac{dy}{dt} &= \frac{\beta xy}{ax+1} - \mu y.\end{aligned} \tag{2}$$

The variables in this model include the growth of *prey* (r), competition between prey individuals (b), environmental carrying capacity (K), prey saturation level (a), predation on prey (α), the maximum value of population reduction of predator due to competition between individuals to obtain prey (β), and natural mortality of predator (μ)

The harvesting element in prey refers to research by Suryanto et al. [13] that examines linear harvesting in prey with a constant effort harvesting type and the presence of feedback control. The model is stated as follows:

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1 - \frac{x}{K} - ay\right) - Hx,\\ \frac{dy}{dt} &= -ey + Cx,\end{aligned} \tag{3}$$

where $x \geq 0$, $y \geq 0$. The variables in this model include the harvest rate (H), natural mortality (e), predation on prey (a), the positive parameter feedback control (C), and the environmental carrying capacity (K).

2.2. Research methods

In the research process, it is necessary to use several methods to achieve research objectives.

a. Linearization

   Linearization is the process of converting a system of nonlinear differential equations into a system of linear differential equations. Linearization is performed to determine the behavior of the system around the equilibrium point. The results of this linearization are presented in the form of a Jacobian matrix. Carl Gustav Jacob Jacobi (1804–1851), a German analyst, introduced the Jacobian matrix. The Jacobian matrix is a matrix whose elements are the first partial derivatives of several functions used to analyze the local stability of a system's equilibrium point [2]. Given a system of nonlinear differential equations expressed as:

$$\frac{dx}{dt} = F(x,y,z),\ \frac{dy}{dt} = G(x,y,z),\ \text{and}\ \frac{dz}{dt} = H(x,y,z). \tag{4}$$

The Taylor series of a real function $f(x)$ can be expressed as follows:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + R_n(x) \tag{5}$$

In eq. (5), all the nonlinear terms and the remainder term $R_n(x)$ is assumed to be $\eta(x)$. Assume that the function in eq. (5) has continuous partial derivatives at the point (x_0, y_0, z_0). Then the Taylor series of the functions $F(x,y,z), G(x,y,z)$ and $H(x,y,z)$ can be written as follows:

$$\begin{aligned} F(x,y,z) &= F(x_0,y_0,z_0) + F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0) \\ &\quad (y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) + \eta_1(x,y,z), \\ G(x,y,z) &= G(x_0,y_0,z_0) + G_x(x_0,y_0,z_0)(x-x_0) + G_y(x_0,y_0,z_0) \\ &\quad (y-y_0) + G_z(x_0,y_0,z_0)(z-z_0) + \eta_2(x,y,z), \\ H(x,y,z) &= H(x_0,y_0,z_0) + H_x(x_0,y_0,z_0)(x-x_0) + H_y(x_0,y_0,z_0) \\ &\quad (y-y_0) + H_z(x_0,y_0,z_0)(z-z_0) + \eta_3(x,y,z), \end{aligned} \tag{6}$$

where $F(x_0,y_0,z_0) = G(x_0,y_0,z_0) = H(x_0,y_0,z_0) = 0$ and $\frac{dx}{dt} = \frac{d(x-x_0)}{dt}$, $\frac{dy}{dt} = \frac{d(y-y_0)}{dt}$, $\frac{dz}{dt} = \frac{d(z-z_0)}{dt}$. Then the equation is reduced to the matrix form below:

$$\frac{d}{dt}\begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix} = \begin{pmatrix} F_x(x_0,y_0,z_0) & F_y(x_0,y_0,z_0) & F_z(x_0,y_0,z_0) \\ G_x(x_0,y_0,z_0) & G_y(x_0,y_0,z_0) & G_z(x_0,y_0,z_0) \\ H_x(x_0,y_0,z_0) & H_y(x_0,y_0,z_0) & H_z(x_0,y_0,z_0) \end{pmatrix} \begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix} + \begin{pmatrix} \eta_1(x,y,z) \\ \eta_2(x,y,z) \\ \eta_3(x,y,z) \end{pmatrix}. \tag{7}$$

Then the Jacobi matrix is obtained as follows:

$$J(x,y,z) = \begin{pmatrix} \frac{\partial F(x_0,y_0,z_0)}{\partial x} & \frac{\partial F(x_0,y_0,z_0)}{\partial y} & \frac{\partial F(x_0,y_0,z_0)}{\partial z} \\ \frac{\partial G(x_0,y_0,z_0)}{\partial x} & \frac{\partial G(x_0,y_0,z_0)}{\partial y} & \frac{\partial G(x_0,y_0,z_0)}{\partial z} \\ \frac{\partial H(x_0,y_0,z_0)}{\partial x} & \frac{\partial H(x_0,y_0,z_0)}{\partial y} & \frac{\partial H(x_0,y_0,z_0)}{\partial z} \end{pmatrix}. \tag{8}$$

b. Eigenvalues and Eigenvectors

Definition 1. [16] If $\mathbf{A}$ is a matrix of size $n \times n$ such that a nonzero vector $\mathbf{x}$ in $\mathbb{R}^n$ is called an eigenvector of $\mathbf{A}$. If $\mathbf{x}$ is a scalar multiple of $\mathbf{x}$, which can be expressed as follows

$$\mathbf{Ax} = \lambda \mathbf{x}. \tag{9}$$

for an arbitrary scalar λ is called the eigenvalue of $\mathbf{A}$ and $\mathbf{x}$ is called the eigenvector of $\mathbf{A}$.

The eigenvalue of a matrix $\mathbf{A}$ of size $n \times n$, then eq. (9) can be rewritten as,

$$\begin{aligned} \mathbf{Ax} &= \lambda \mathbf{Ix}. \\ \mathbf{Ax} - \lambda \mathbf{Ix} &= 0. \\ (\mathbf{A} - \lambda \mathbf{I})\mathbf{x} &= 0. \end{aligned} \tag{10}$$

Where I is the identity matrix. In eq. (10) there is $\mathbf{x}$ which has a non-zero solution, so it can be obtained if and only if,

$$det(\mathbf{A} - \lambda \mathbf{I}) = 0. \tag{11}$$

Equation (11) is the characteristic equation of the matrix $\mathbf{A}$ and the scalar λ is the eigenvalue of the matrix $\mathbf{A}$.

This research is a literature study that discusses concepts and theories through various library sources related to the problem, taken through numerical calculations using Maple 21 software. The following is the research design:

1. Literature Study
   At this stage, the author conducted a literature study of ecological phenomena related to interactions and behavior between populations based on sources from scientific journals, books, encyclopedias, and previous research articles related to the problem being discussed. Examples include analysis of existing theories and previous research results.
2. Model Construction
   In this stage, the author constructed a model using several literature sources on the predator-prey system, which was then developed into a three-species Modified Leslie Gower model with prey harvesting and a Holling Type II functional response.
3. Determining the Equilibrium Point
   In this stage, the author determined the equilibrium point of the three-species Modified Leslie Gower predator-prey model with prey harvesting using the Holling Type II functional response. The equilibrium point solution was obtained by zeroing the right-hand and left-hand sides of each equation.
4. Equilibrium Point Stability Analysis
   In this stage, the author analyzed the model to evaluate the stability of the equilibrium point. This model is called a nonlinear differential equation, so it is necessary to first linearize the model equations. After constructing the Jacobian matrix, the next step is to determine the stability of the equilibrium point using the eigenvalue criterion or the Routh-Hurwitz criterion.
5. Numerical Simulation
   In this stage, the author conducted numerical simulations to model and analyze the stability behavior around the equilibrium point in the predator-prey model using Python. This simulation was also used to confirm the suitability of the analytical results of the predator-prey model stability with the Holling Type II Response Function.
6. Conclusion
   In this stage, the author drew conclusions from the results of the stability and numerical analysis of the behavior of the modified Leslie Gower predator-prey model of three species with harvesting on the prey using the Holling Type II Response Function.

3. Analytical Results

3.1. Model Construction

The predator-prey population interaction model is constructed as follows:

- Population growth rate of prey (field rat)
  The population of prey is denoted as x at time t. The logistic population growth is $rx\left(1-\frac{x}{K}\right)$ with an intrinsic growth rate of r and an environmental carrying capacity of K. The rat population also experiences losses due to predation by predators of $\frac{\alpha xy}{1+ax}$. This term follows a Holling type II functional response, which takes into account the limited ability of predators to handle prey. The parameter α represents the predator attack rate, a indicates the predation saturation level, and the harvesting activity of prey is H. The growth rate of prey can be written as follows:

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \frac{\alpha xy}{1+ax} - Hx. \tag{12}$$

- Population growth rate of predator (jali snake)
  The population of predator is expressed as y at time t. The population growth of predator is derived from the consumption of *prey* of $\frac{\eta xy}{1+ax}$, with η being the biomass conversion efficiency

and a the predation saturation level. The jali snake population also experienced a decline due to natural mortality of μ. In addition, there was predation by jali snakes on owls of $\frac{\beta yz}{1+by}$ with an owl attack rate of β and a predation saturation level of b. The population growth rate of a predator can be written as follows:

$$\frac{dy}{dt} = \frac{\eta xy}{1+ax} - \mu y - \frac{\beta yz}{1+by}. \tag{13}$$

- Population growth rate of a super predator (owl)
  The population of a super predator is denoted as z at time t. The growth of the owl population is assumed to follow the Modified Leslie–Gower form of $sz\left(1-\frac{z}{e+y}\right)$, where s is the growth rate of the owl, e is the environmental contribution in providing protection for the predator, and $e+y$ represents the environmental carrying capacity, which depends on the amount of food available to the predator (jali snake) population. The population growth rate of a super predator can be written as follows:

$$\frac{dz}{dt} = sz\left(1-\frac{z}{e+y}\right). \tag{14}$$

3.2. Equilibrium Point

The equilibrium point in the system of equations (1), (2), (3) can be obtained by making the right-hand side of each equation equal to zero, then:

$$\begin{aligned}
\left[r\left(1-\frac{x}{K}\right) - \frac{\alpha y}{1+ax} - H\right]x &= 0,\\
\left[\frac{\eta x}{1+ax} - \mu - \frac{\beta z}{1+by}\right]y &= 0,\\
\left[s\left(1-\frac{z}{e+y}\right)\right]z &= 0.
\end{aligned}$$

Thus, we obtain

a) The equilibrium Point $E_1 = (0,0,0)$ indicates that the prey, predator and super predator populations are extinct, due to the lack of inter specific interactions that could sustain each other.
b) The equilibrium Point $E_2 = \left(-\frac{K(H-r)}{r},0,0\right)$ indicates that the predator and super predator populations are extinct, while the prey population does not become extinct or exists if it meets the condition $H-r<0$, that is, $H<r$.
c) The equilibrium Point $E_3 = \left(-\frac{K(H-r)}{r},0,e\right)$ indicates that the predator population is experiencing extinction, while the prey and super predator populations are not experiencing extinction or exist if they meet the conditions $H-r<0$, namely $H<r$.
d) The equilibrium Point $E_4 = \left(-\frac{\mu}{a\mu-\eta}, \frac{\eta(-rka\mu+rk\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2},0\right)$ states that the population of super predator is extinct, while the population of prey and predator is not extinct or exists if it meets the conditions $a\mu<\eta$ and $\frac{\eta(-rka\mu+rk\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2}<0$.
e) The equilibrium Point $E^* = (x^*,y^*,z^*)$ indicates the state of the prey, predator, and super predator populations, whether they are not extinct or exist.

3.3. Equilibrium Point Stability Analysis

Stability analysis is obtained by examining the eigenvalues of the system's linearization results using the Jacobian matrix. The stability of the system in equations (1), (2), and (3) is obtained from five equilibrium points with the following Jacobian matrix:

$$J(x,y,z)=\begin{pmatrix} r-\dfrac{2rx}{K}-\dfrac{\alpha y}{(1+ax)^2}-H & -\dfrac{\alpha x}{1+ax} & 0 \\ \dfrac{\eta y}{(1+ax)^2} & \dfrac{\eta x}{1+ax}-\mu-\dfrac{\beta z}{(1+by)^2} & \dfrac{-\beta y}{1+by} \\ 0 & \dfrac{sz^2}{(e+y)^2} & s-\dfrac{2sz}{e+y} \end{pmatrix}. \tag{15}$$

To determine the dynamic behavior of a system of equations analytically, a local stability analysis can be performed using the following eigenvalues.

1) Stability of Equilibrium Point $E_1=(0,0,0)$
   Substituting $x=0,\ y=0,\ z=0$ into the Jacobian matrix, we obtain

$$JE_1=\begin{bmatrix} r-H & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & 0 & s \end{bmatrix} \tag{16}$$

   The characteristic equation of the Jacobian matrix JE_1 with $det(JE_1)-\lambda I=0$ is $(r-H-\lambda)(-\mu-\lambda)(s-\lambda)=0$. Then the eigenvalues $\lambda_1=r-H,\ \lambda_2=-\mu$, and $\lambda_3=s$ are obtained, so E_1 is saddle point, unstable.

2) Stability of Equilibrium Point $E_2=\left(-\dfrac{K(H-r)}{r},0,0\right)$
   Substituting $x=-\dfrac{K(H-r)}{r},\ y=0,\ z=0$ into the Jacobian matrix, we obtain

$$JE_2=\begin{bmatrix} H-r & \dfrac{\alpha K(H-r)}{r-aK(H-r)} & 0 \\ 0 & -\dfrac{\eta K(H-r)}{r-aK(H-r)}-\mu & 0 \\ 0 & 0 & s \end{bmatrix} \tag{17}$$

   The characteristic equation of the Jacobian matrix JE_2 with $det(JE_2)-\lambda I=0$ is

$$(H-r-\lambda)\left(-\frac{\eta K(H-r)}{r-aK(H-r)}-\mu-\lambda\right)(s-\lambda)=0.$$

   Then the eigenvalues $\lambda_1=H-r$, $\lambda_2=-\dfrac{\eta K(H-r)}{r-aK(H-r)}-\mu$, and $\lambda_3=s$, so that E_2 is saddle point, unstable.

3) Stability of Equilibrium Point $E_3=\left(-\dfrac{K(H-r)}{r},0,e\right)$
   Substituting $x=-\dfrac{K(H-r)}{r},\ y=0,\ z=e$ into the Jacobian matrix, we obtain

$$JE_3=\begin{bmatrix} H-r & \dfrac{\alpha K(H-r)}{r-aKH+aKr} & 0 \\ 0 & -\dfrac{\eta K(H-r)}{r-aK(H-r)}-\mu-\beta e & 0 \\ 0 & s & -s \end{bmatrix} \tag{18}$$

The characteristic equation of the Jacobian matrix JE_3 with $det(JE_3) - \lambda I = 0$ is

$$(H-r-\lambda)\left(-\frac{\eta K(H-r)}{r-aK(H-r)}-\mu-\beta e-\lambda\right)(-s-\lambda)=0.$$

Then the eigenvalues $\lambda_1 = H-r, \lambda_2 = -\frac{\eta K(H-r)}{r-aK(H-r)} - \mu - \beta e$, and $\lambda_3 = -s$, so E_3 is saddle point, unstable.

4) Equilibrium Point Stability $E_4 = \left(-\frac{\mu}{a\mu-\eta}, \frac{\eta(-rKa\mu+rK\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2}, 0\right)$
Substituting $x = -\frac{\mu}{a\mu-\eta}$, $y = \frac{\eta(-rKa\mu+rK\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2}$, $z = 0$ into the Jacobi matrix (15), then we obtain

$$JE_4 = \begin{bmatrix} A_{11} & A_{21} & 0 \\ A_{12} & A_{22} & A_{32} \\ 0 & 0 & A_{33} \end{bmatrix}, \tag{19}$$

where

$$A_{11} = r - \frac{2r(-\frac{\mu}{a\mu-\eta})}{K} - \frac{\alpha(\frac{\eta(-rKa\mu+rK\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2})}{(1+a(-\frac{\mu}{a\mu-\eta}))^2} - H.$$

$$A_{12} = \frac{\eta(\frac{(-rKa\mu+rK\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2})}{(1+a(-\frac{\mu}{a\mu-\eta}))^2}$$

$$A_{21} = \frac{-\alpha(-\frac{\mu}{a\mu-\eta})}{(1+a(-\frac{\mu}{a\mu-\eta}))^2}$$

$$A_{22} = -\frac{\eta(-\frac{\mu}{a\mu-\eta})}{(1+a(-\frac{\mu}{a\mu-\eta}))} - \mu$$

$$A_{32} = \frac{-\beta(\frac{\eta(-rKa\mu+rK\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2})}{(1+b(\frac{\eta(-rKa\mu+rK\eta-r\mu-KHa\mu+KH\eta)}{K\alpha(a\mu-\eta)^2}))}$$

$$A_{33} = s$$

The characteristic equation of the Jacobian matrix JE_4 with $det(JE_4) - \lambda I = 0$ is $\lambda_1 = A_{33}$, for λ_2 and λ_3 is obtained from $\lambda^2 - T_1\lambda + D_1 = 0$. With Trace $= T_1 = A_{11} + A_{22}$ and Determinant $= D_1 = A_{11}A_{22} - A_{12} + A_{21}$. Then the eigenvalue is obtained If $\lambda_1 < 0$ and $\lambda_2, \lambda_3 \in \mathbb{R}$ with $\lambda_2, \lambda_3 < 0$, so that E_4 is node, asymptotically stable.

5) Stability of Equilibrium Point $E^* = (x^*, y^*, z^*)$
Substituting $x = x^*, y = y^*, z = z^*$ into the Jacobi matrix, we obtain

$$JE^* = \begin{bmatrix} B_{11} & B_{21} & 0 \\ B_{12} & B_{22} & B_{32} \\ 0 & B_{23} & B_{33} \end{bmatrix}. \tag{20}$$

where

$$B_{11} = r - \frac{2rx^*}{K} - \frac{\alpha y^*}{(1+ax^*)^2} - H, \qquad B_{12} = \frac{\eta y^*}{(1+ax^*)^2},$$

$$B_{21} = \frac{-\alpha y^*}{1+ax^*}, \qquad B_{22} = \frac{\eta x^*}{1+ax^*} - \mu - \frac{\beta z^*}{(1+by^*)^2},$$

$$B_{23} = \frac{sz^*}{(e+y^*)^2}, \qquad B_{32} = \frac{-\beta y^*}{1+by^*},$$

$$B_{33} = s - \frac{2sz^*}{e+y^*}.$$

The characteristic equation obtained is $\lambda^3 + a_1\lambda^2 + a_1\lambda + a_1 = 0$, where

$$\begin{aligned} a_1 &= -(B_{11}B_{22}B_{33}),\\ a_2 &= B_{11}B_{22} + B_{11}B_{33} + B_{22}B_{33} - B_{23}B_{32} - B_{12}B_{21},\\ a_3 &= -B_{11}B_{22}B_{33} + B_{11}B_{23}B_{32} + B_{12}B_{21}B_{33}. \end{aligned}$$

Based on the Routh Hurwitz criterion, all eigenvalues will be negative if $a_1 > 0$, $a_3 > 0$, $a_1a_2 > a_3$ then E^* is asymptotically stable.

4. Numerical Results

Numerical simulations were used to determine changes in dynamic behavior around equilibrium points and to verify the stability analysis results of the predator-prey model using Python software.The parameter values used in the simulation are presented in the following table:

Table 1. Parameter Values

Parameters	Values	Sources
r	2.0	[17]
α	0.6	[18]
a	0.5	[19]
H	0.5	Assumptions
η	0.8	[18]
μ	0.1	[19]
β	0.3	[19]
b	0.6	[20]
s	1	[20]
e	0.5	[20]

Table 2. Stability with a value of $K = 0.3$

Equilibrium Point	Eigen Value	Stability Properties
$E_1(0,\ 0,\ 0)$	$\lambda_1 = 1.5$, $\lambda_2 = -0.1$, $\lambda_3 = 1$	Saddle point, unstable
$E_2(0.225,\ 0,\ 0)$	$\lambda_1 = -1.5$, $\lambda_2 = 0.06179$, $\lambda_3 = 1$	Saddle point, unstable
$E_3(0.225,\ 0,\ 0.5)$	$\lambda_1 = 1.5$, $\lambda_2 = -0.1$, $\lambda_3 = 1$	Node, asymptotically stable
$E_4(0.13333,\ 1.08641,\ 0)$	$\lambda_1 = -0.77695$, $\lambda_2 = -0.07373$, $\lambda_3 = 1$	Saddle point, unstable

In this study, numerical simulations were performed by substituting the parameter values in Table 1 to determine the stability of the system solution at different environmental carrying capacity parameter values (K). The dynamic behavior was displayed through phase portraits using Python software based on the calculation of the stability conditions, namely $K < \dfrac{r(\mu+\beta e)}{(r-H)(\eta - a\mu - a\beta e)}$ with the limit $K_0 = 0.4938$. Numerical simulations were performed by taking the values $K = 0.3 < K_0$ and $K = 0.6 > K_0$.

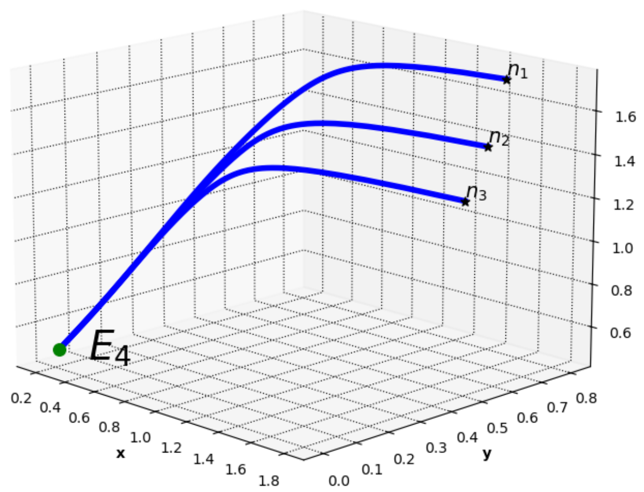

Figure 1. Phase Portrait when $K = 0.3$ with 3 different initial values

a) Numerical simulation with the value of $K = 0.3$
The simulation results with the parameter value $K = 0.3$ are shown in full in Table 2. Based on Table 2, it shows that there are four existing equilibrium points, namely E_1, E_2, E_3, E_4 while the equilibrium point that does not exist is E_5 because it has a negative value. There is one asymptotically stable equilibrium point with the type of equilibrium in the form of a Node, namely at the equilibrium point $E_4(0.225, 0, 0.5)$. The simulation results with parameter values in Table 2 when $K = 0.3$ are illustrated with the python software in The simulation results with parameter values in Table 1 when $K = 0.3$ is given by fig. 1.Based on fig. 1 shows a phase portrait with three different initial values, namely $[1.5, 0.8, 1.7]$, $[1.75, 0.6, 1.5]$, and $[1.8, 0.5, 1.3]$ towards point $E_4(0.225,\ 0,\ 0.5)$. Therefore, it can be concluded that the stability of the equilibrium point E_4 is an asymptotically stable node, meaning that the populations of field mice, snakes, and owls exist or can coexist.
The following is a phaseportrait of the time series when $K = 0.3$. Based on fig. 2 shows that

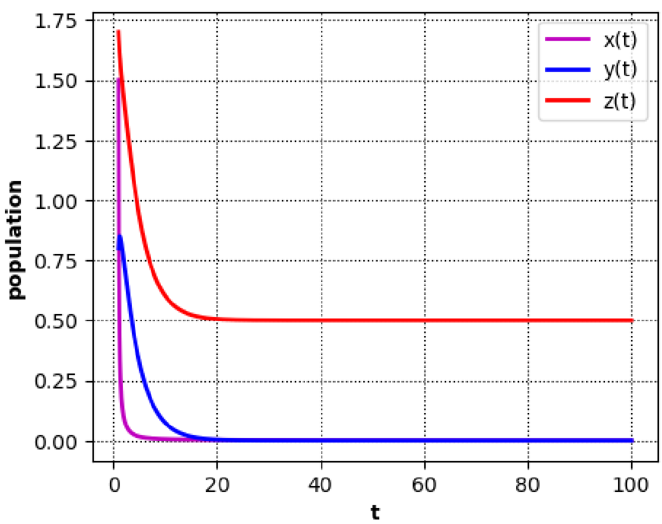

Figure 2. Time series when $K = 0.6$ with $t = 100$

when the environmental carrying capacity rate is $K = 0.3$ with initial values $[1.5, 0.8, 1.7]$ it is asymptotically stable towards the equilibrium point $E_4(0.225,\ 0,\ 0.5)$. At the initial value of the rice field rat population of 1.5 individuals towards the equilibrium point $x = 0.225$. The initial value of the population of the snake jali is 0.8 individuals towards the equilibrium point $y = 0$. Then the initial value of the population of the snake jali is 1.7 individuals towards the equilibrium point $z = 0.5$. fig. 2 shows the environmental carrying capacity carried out by the rice field rat of $K = 0.11$ with the initial population value $[1.5, 0.8, 1.7]$ stating that the population of rice field rats, snake jali, and owls can survive side by side or not experience extinction.

b) Numerical simulation with $K = 0.6$
The simulation results with the parameter value $K = 0.6$ are shown in full in the following table:

Table 3. Stability with a value of $K = 0.3$

Equilibrium Point	Eigen Value	Stability Properties
$E_1(0,\ 0,\ 0)$	$\lambda_1 = 1.5,\ \lambda_2 = -0.1,\ \lambda_3 = 1$	Saddle point, unstable
$E_2(0.44999,\ 0,\ 0)$	$\lambda_1 = -1.5,\ \lambda_2 = 0.19387,\ \lambda_3 = 1$	Saddle point, unstable
$E_3(0.44999,\ 0,\ 0.5)$	$\lambda_1 = 1.5,\ \lambda_2 = 0.04387,\ \lambda_3 = -1$	Saddle point, unstable
$E_4(0.13333,\ 1.87654,\ 0)$	$\lambda_1 = -0.18923 + 0.25129I$ $\lambda_2 = -0.18923 - 0.251291$ $\lambda_3 = 1$	Saddle point, unstable
$E^*(0.426,\ 0.16172,\ 0.66172)$	$\lambda_1 = -1.39378,\ \lambda_2 = -0.04384,\ \lambda_3 = -0.95231$	Node, asymptotically stable

Based on Table 3, it shows that there are four existing equilibrium points, namely E_1, E_2, E_3, E_4, E^*. There is one asymptotically stable equilibrium point with the type of equilibrium in the form of a Node, namely at the equilibrium point $E^*(0.426,\ 0.16172,\ 0.66172)$. The simulation results with parameter values in Table 1 when $K = 0.6$ are illustrated with the Python software below.

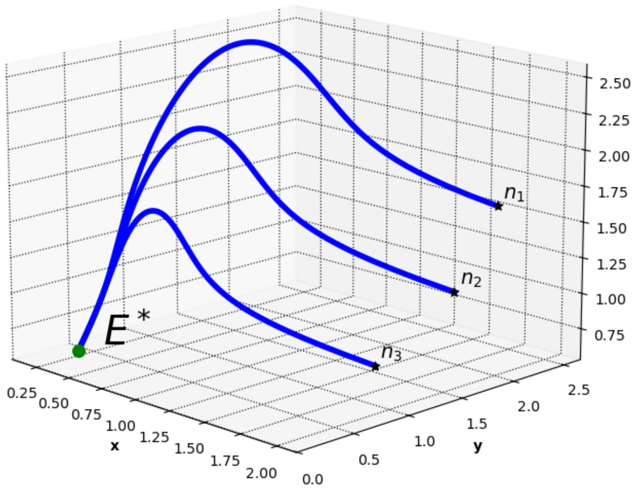

Figure 3. Phase Portrait when $K = 0.6$ with 3 different initial values

Based on fig. 3, it shows a phase portrait with 3 different initial values, namely $[0.9, 2.6, 3.2]$, $[0.7, 2.4, 2.9]$, $[0.082, 2.45, 3.1]$ towards the point $E^*(0.426,\ 0.16172,\ 0.66172)$. Therefore, it can be concluded that the stability of the equilibrium point E^* is in the form of an asymptotically stable node, meaning that the populations of field mice, snakes, and owls exist or can coexist.
The following is a numerical simulation of the time series $K = 0.6$. Based on fig. 4, it shows that when the environmental carrying capacity rate is $K = 0.6$ with initial values $[1.8, 2.3, 1.6]$, it is asymptotically stable towards the equilibrium point $E^*(0.426, 0.16172,\ 0.66172)$. At the initial value, the population of field mice is 1.8 individuals towards the equilibrium point $x = 0.426$. The population of the initial value of the jali snake is 2.3 individuals towards the equilibrium point $y = 2.537283$. Then the population of the initial value of the owl is 1.6 individuals towards the

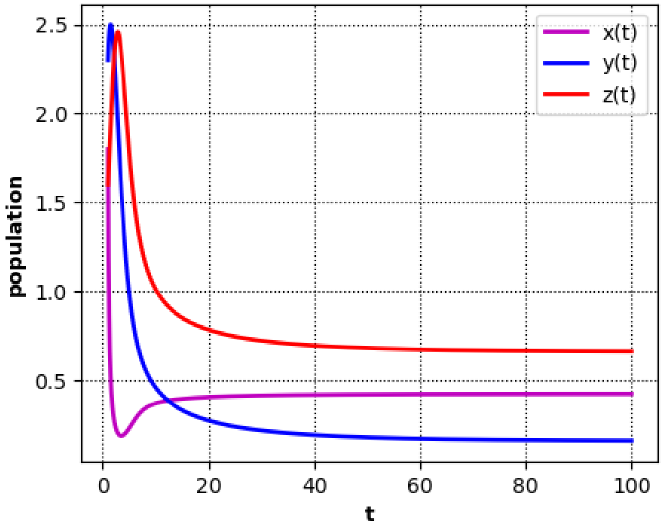

Figure 4. Time series when $K = 0.6$ with $t = 100$

equilibrium point $z = 3.037283.5$. In the time series fig. 4, the environmental carrying capacity of the rice field mice is $K = 0.6$ with an initial population value of $[1.8, 2.3, 1.6]$, which states that the population of rice field mice, jali snakes, and owls can survive side by side or not experience extinction.

5. Conclusion

A modified Leslie-Gower model of three species with constant effort harvesting on prey and Holling type II functional response was constructed with phenomena that occur in rice field ecosystems, one of which is the phenomenon of rice field rats, snakes, and owls. Uncontrolled rice field rat attacks can cause significant losses. Consequently, additional strategies are needed to control the rat population, one of which is through a harvesting approach. Analysis of the equilibrium points was carried out to determine the local stability conditions using the Jacobian approach and the Routh-Hurwitz criterion with the acquisition of five existing points. Stability analysis using the Jacobian matrix showed that the equilibrium points are locally asymptotically stable nodes under certain conditions. Numerical simulations were carried out to determine the suitability of the analysis results using software Maple and Python. The results of the numerical simulations showed that differences in the environmental carrying capacity parameter (K) affected changes in the system solution. Thus, changes in the value of K do not always result in the same stability, because the stability of the system depends on the sensitivity parameters to the overall dynamic structure. These findings provide a basis for managing biological resources, especially in controlling harvesting to ensure that populations remain balanced.

Supplementary Information

Acknowledgements. We are incredibly grateful to the editors and reviewers for their helpful comments and invaluable assistance in improving this work.

Funding. This research received no external funding.

Author Contributions. **Nabila Kholifatul Yuniar Eghary:** Conceptualization, Methodology, Writing-Review & Editing. **Dian Savitri:** Conceptualization, Validation, Methodology, Writing-Review & Editing, Project Administration, Supervision, Funding Acquisition, Resources. All authors read and approved the final manuscript.

Conflict of interest. The authors declare no conflict of interest.

Data availability. All data presented are hypothetical and cited from prior publications.

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