

Dynamics of Alcohol Consumption Model with Public Awareness and Saturated Incidence

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Abstract

This article develops a mathematical model for studying the impact of public awareness and intervention strategies on alcohol consumption patterns. To protect populations from addiction, we are focusing on educational campaigns and interventions. Alcohol consumption cases decrease as the model variable awareness susceptible class is increased through awareness campaigns and interventions. The nonnegativity and boundedness of the model's solutions are analyzed. A qualitative analysis of the model's equilibrium points and the alcohol reproductive number, $\mathcal{R}_0$, was performed. Global stability was analyzed for alcohol consumption at the positive equilibrium point via a suitable Lyapunov function. When the alcohol reproductive number ($\mathcal{R}_0$) is less than one, the alcohol-free equilibrium is globally asymptotically stable; otherwise, it is unstable. Although educational campaigns protect vulnerable people, their impact on the model is substantial. The simulation shows that intervention directly and drastically reduces the target alcoholic population.

Keywords : Alcohol consumption model · Public awareness · Saturated incidence · Stability

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1. Introduction

Across diverse scientific fields, mathematical modeling has been essential for decades in simulating complex phenomena, gaining understanding, and forecasting future outcomes [1, 2]. Mathematical models can be used to study different problems, their causes, and contributing factors [3, 4]. Many scholars have mathematically modeled the phenomenon, each using a variety of mathematical techniques. The contagion theory and the social contexts involved explain the spread of this social behavior. Associating with people who drink, whether heavily or lightly, can significantly impact one's life. Thus, the same modeling techniques used for other diseases can also apply to alcohol-related disorders, including alcoholism [5].

Since misuse of alcohol is a real-world problem. Understanding how drug abuse develops and its destructive impact on human life is the focus of researchers who use a range of analytical tools. The

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area of mathematical modeling is vital for analyzing and researching real-world problems. For example, mathematical models play a crucial role in controlling various diseases. Although past research has successfully modeled infectious diseases to predict the effects of epidemics, the world continues to grapple with managing such crises due to natural or human-made factors. Global risk management requires the expertise of mathematicians [6]. Many researchers find mathematical modeling of alcohol abuse and other behavioral changes fascinating. This approach likely gained traction because of the parallels between the spread of alcoholism and infectious diseases [7]. Most models of alcoholism and its consequences adapt the basic *SIR* model with substantial modifications [7, 8].

Modeling efforts have increasingly examined how awareness campaigns influence the behavior of individuals. These models account for how awareness affects the adoption of preventive measures for various problems [9–13]. Social media awareness campaigns are a proven, cost-effective way to reach individuals [9, 13]. Awareness campaigns can also target specific groups, offering solutions to their particular problems [13]. Recently, several researchers have considered the individual consciousness factor in a mathematical model in various cases, for example Malaria spread [9, 11], COVID-19 spread [10], HIV/AIDS spread [14], Dengue spread [15], Rabies spread [16], Financial problem [12], Election problem [13], Tobacco [17], including Alcohol problems.

For alcoholism dynamics, several researchers have investigated the success of individual awareness strategies. In 2015, Ma et al. [18] found that the model analysis shows that, though awareness programs cannot eradicate alcohol problems, they are adequate measures for controlling alcohol problems. Next, the spread of alcohol addiction is controlled by creating awareness among the people, as investigated by Ramamoorthi and Muthukrishnan in [19]. Based on results from Muchika et al. in [20], the number of light and heavy drinkers rises further because of poor implementation of media programs; thus, the need for the government and other stakeholders to plan suitable policies in targeting media awareness as an intervention measure in combating the rampant alcoholism cases in the community. Kunwar and Verma in [21] show that an awareness program is an effective measure in controlling the heavy drinking problem.

Related to the alcohol dynamics, In 2021, Din and Li [5] proposed a stochastic model of alcohol consumption with three groups of population: non-consumers, social drinkers, and risk-drinkers. Next, Sher et al. [6] analyzed the dynamics of the alcohol model, including potential drinkers, moderate alcohol abusers, heavy alcohol abusers, heavy alcohol abusers with complications, and recovery groups. Considering the compulsory isolation treatment for the prevalence of alcoholism is proposed by Mayengo [7]. Currently, Sivashankar et al. [8] analyzed the stabilities of a fractional model considering susceptible individuals, exposed individuals, addictive individuals, depressed individuals, and recovered individuals. The forms of interaction in the various studies referred to still do not take the saturation factor into account. Recently, some research related to mathematical models has considered the saturation factor of an incidence [1, 22–26]. This is very interesting to study in the alcohol problem.

Motivated by previous works, we modified a mathematical model for alcohol consumption by Sivashankar et al. [8] with public awareness and saturated incidence. This becomes necessary to gain more insights into the population dynamics of alcohol consumption and curtail alcohol addiction. Organization of the paper: The model of alcohol consumption in Section 2 is designed with dynamical plausibility in mind. We show that the model preserves the properties of positivity, boundedness, existence of equilibrium points, alcohol reproductive number, and global stability in Section 3. We derive some numerical simulations to support our analysis in Section 4. In Section 5, we present our conclusion.

2. Model Formulation for Alcohol Consumption

Six population categories are used in this model. The unaware susceptible individuals (S_u) comprise those who do not drink alcohol but are potential future drinkers. Education-based abstinence

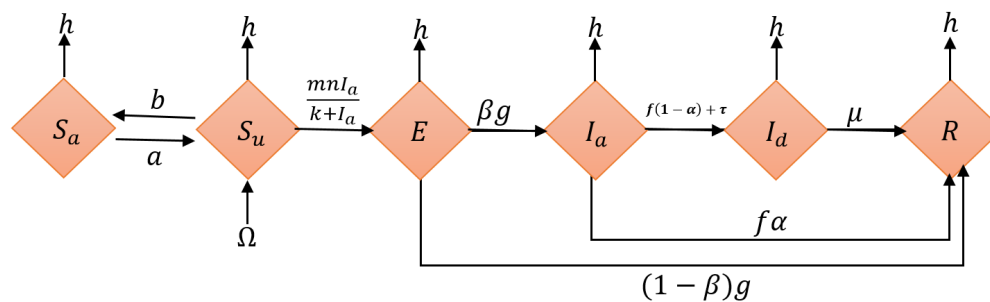


Figure 1. Scheme of alcohol consumption

defines susceptible individuals (S_a) concerning alcohol consumption. Individuals in the group (E) are at risk for alcohol addiction because of their occasional alcohol use. Individuals suffering from alcohol addiction (I_a) are characterized by high alcohol consumption. Individuals experiencing depression resulting from alcohol consumption are categorized within the group I_d . Recovered individuals (R) are those who have finished treatment and are no longer addicted. The total number of individuals, denoted by K , is given as $K = S_u + S_a + E + I_a + I_d + R$. Ω represents the rate at which susceptible individuals join the population. Alcohol awareness programs for at-risk individuals increase awareness at a rate of a . Educated individuals may gradually return to normal alcohol consumption at a rate of b . Peers with alcohol addictions can influence unaware individuals, leading to alcohol use at a contact rate of m and a transmission rate of n . This moves them into the exposed class. Alcohol use causes a fraction β of the exposed population to become addicted at rate g , while the remaining fraction $(1 - \beta)$ recovers through treatment at the same rate g . An intervention affects the incident process at a rate k . Addiction relapse causes depression at rate τ ; those leaving treatment or needing a second dose also experience depression at rate $(1 - \alpha)f$. Treatment and education help depressed individuals recover at rate μ , with successful treatment recovery at rate $f\alpha$. The overall death rate is h . Next, the scheme of alcohol consumption is illustrated in Figure 1.

Therefore, based on the interaction diagram in Figure 1, the mathematic model of alcohol consumption can be constructed:

$$\begin{aligned}
 \frac{dS_u}{dt} &= \Omega + bS_a - \frac{mnI_aS_u}{k + I_a} - aS_u - hS_u, \\
 \frac{dS_a}{dt} &= aS_u - (b + h)S_a, \\
 \frac{dE}{dt} &= \frac{mnI_aS_u}{k + I_a} - (g + h)E, \\
 \frac{dI_a}{dt} &= \beta gE - (\tau + f + h)I_a, \\
 \frac{dI_d}{dt} &= f(1 - \alpha)I_a + \tau I_a - (\mu + h)I_d, \\
 \frac{dR}{dt} &= (1 - \beta)gE + \mu I_d + f\alpha I_a - hR,
 \end{aligned} \tag{1}$$

with initial condition

$$S_u > 0, S_a > 0, E \geq 0, I_a \geq 0, I_d \geq 0, R \geq 0. \tag{2}$$

3. Analytical Results

3.1. Positivity and Boundedness

Theorem 1. Any solution $(S_u, S_a, E, I_a, I_d, R)$ of system (1) with non-negative initial conditions (2) is positive for all time $t > 0$.

Proof. From alcohol model in system (1), we have

$$\begin{aligned}
 \left. \frac{dS_u}{dt} \right|_{S_u=0} &= \Omega + bS_a > 0, \\
 \left. \frac{dS_a}{dt} \right|_{S_a=0} &= aS_u > 0, \\
 \left. \frac{dE}{dt} \right|_{E=0} &= \frac{mnI_a S_u}{k + I_a} \geq 0, \\
 \left. \frac{dI_a}{dt} \right|_{I_a=0} &= \beta gE \geq 0, \\
 \left. \frac{dI_d}{dt} \right|_{I_d=0} &= f(1 - \alpha)I_a + \tau I_a \geq 0, \\
 \left. \frac{dR}{dt} \right|_{R=0} &= (1 - \beta)gE + \mu I_d + f\alpha I_a \geq 0.
 \end{aligned} \tag{3}$$

The preceding calculation demonstrates that all rates are non-negative within the boundaries of the positive region $\mathbb{R}_+^6$. Thus, solutions are confined to the positive region whenever the system's initial state is in the non-negative region $\mathbb{R}_+^6$. ■

Theorem 2. All solution of system (1) are bounded for all $t \in [0, t_0]$

Proof. Since we add all equations in (1), we get

$$K(t) = S_u(t) + S_a(t) + E(t) + I_a(t) + I_d(t) + R(t). \tag{4}$$

Next, By substituting system (1) into the derivative results from eq. (4), we get

$$\frac{dK}{dt} \leq \Omega - hK \tag{5}$$

Thusly,

$$0 \leq \limsup_{x \rightarrow \infty} K(t) \leq \frac{\Omega}{h}, \tag{6}$$

so all solutions of system (1) are ultimately bounded for all $t \in [0, t_0]$. So, the region:

$$\Sigma = \left\{ (S_u, S_a, E, I_a, I_d, R) \in \mathbb{R}_+^6 : 0 \leq K(t) \leq \frac{\Omega}{h} \right\}.$$

3.2. Equilibrium Points and Alcohol Reproductive Number

System (1) always had an alcohol-free equilibrium, which is given by

$$A_0 = \left(\frac{\Omega(b+h)}{h(a+b+h)}, \frac{\Omega a}{h(a+b+h)}, 0, 0, 0, 0 \right) \tag{7}$$

To determine reproductive number $\mathfrak{R}_0$, from model (1) we have the $\mathcal{F}$ transition and $\mathcal{V}$ transmission matrices, where

$$\mathcal{F} = \begin{pmatrix} \frac{mnI_a S_u}{k+I_a} \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathcal{V} = \begin{pmatrix} (g+h)E \\ (\tau+f+h)I_a - \beta gE \\ (\mu+h)I_d - f(1-\alpha)I_a - \tau I_a \end{pmatrix}.$$

Next, the reproductive number is given by

$$F = \begin{pmatrix} 0 & \frac{mn\Omega(b+h)}{kh(a+b+h)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} (g+h) & 0 & 0 \\ -\beta g & (\tau+f+h) & 0 \\ 0 & -f(1-\alpha)-\tau & (\mu+h) \end{pmatrix}.$$

Further, $\mathfrak{R}_0$ is a spectral radius of FV^{-1} is

$$\mathfrak{R}_0 = \frac{\Omega(b+h)mn\beta g}{(g+h)(\tau+f+h)kh(a+b+h)}. \quad (8)$$

Next, alcohol consumption equilibrium, which is given by

$$A_* = (S_u^*, S_a^*, E^*, I_a^*, I_d^*, R^*) \quad (9)$$

where

$$\begin{aligned} S_u^* &= \frac{(b+h)(\Omega\beta g + gk(f+\tau) + hk(f+g+h+\tau))}{g\beta(mn(b+h) + h(a+b+h))}, \\ S_a^* &= \frac{a(\Omega\beta g + gk(f+\tau) + hk(f+g+h+\tau))}{g\beta(mn(b+h) + h(a+b+h))}, \\ E^* &= \frac{mnS_u^*}{g+h} - \frac{\tau+f+h}{\beta g}k, \\ I_a^* &= \frac{(g+h)E^*k}{mnS_u^* - (g+h)E^*}, \\ I_d^* &= \frac{(f(1-\alpha) + \tau)I_a^*}{\mu+h}, \\ R^* &= \frac{f\alpha I_a^* + \mu I_d^* + (1-\beta)gE^*}{h}. \end{aligned}$$

3.3. Stability of Equilibrium Points

Theorem 3. An alcohol-free equilibrium A_0 of the system (1) is globally asymptotically stable for $\mathfrak{R}_0 < 1$.

Proof. Define the Lyapunov function

$$L_0 = k_1 E + k_2 I_a. \quad (10)$$

where $k_1 = \beta g(b+h)$ and $k_2 = (g+h)(a+b+h)$. The derivative of L_0 with respect to t is

$$\begin{aligned} \frac{dL_0}{dt} &= k_1 \frac{dE}{dt} + k_2 \frac{dI_a}{dt} \\ &= k_1 \left(\frac{mnI_a S_u}{k + I_a} - (g+h)E \right) + k_2 (\beta gE - (\tau+f+h)I_a) \\ &= \left(k_1 \frac{mnS_u}{k + I_a} - k_2(\tau+f+h) \right) I_a \\ &\leq \left(k_1 \frac{mnS_u}{k} - k_2(\tau+f+h) \right) I_a \end{aligned} \quad (11)$$

Based on Theorem 2, if the solution of system (1) is bounded by $\frac{\Omega}{h}$ as $t \rightarrow \infty$ then the alcohol-free point is bounded by $\frac{\Omega}{h} = S$. Next, we get

$$\frac{dL_0}{dt} \leq k_2(\tau+f+h)(\mathfrak{R}_0 - 1)I_a \quad (12)$$

Clearly, if $\mathfrak{R}_0 < 1$, then $\frac{dL_0}{dt} < 0$. It led to the alcohol-free point (A_0) being globally asymptotically stable. ■

Theorem 4. If $\mathfrak{R}_0 > 1$, the alcohol consumption equilibrium A_* is exists and globally asymptotically stable, while when $\mathfrak{R}_0 < 1$ the A_* is unstable.

Proof. The global stability of system (1) will be analyzed at point A_* when the condition of A_* exists, and $\mathfrak{R}_0 > 1$ are met. The definition and derivation of the Lyapunov function L_* are as follows:

$$\frac{dL_*}{dt} = \frac{1}{2} [(S_u - S_u^*) + (S_a - S_a^*) + (E - E^*) + (I_a - I_a^*) + (I_d - I_d^*) + (R - R^*)]^2 \quad (13)$$

By differentiating function L_* in eq. (13) with respect to time for System (1), we get

$$\frac{dL_*}{dt} = [(S_u - S_u^*) + (S_a - S_a^*) + (E - E^*) + (I_a - I_a^*) + (I_d - I_d^*) + (R - R^*)] \frac{dK}{dt} \quad (14)$$

Since $[S_u + S_a + E + I_a + I_d + R] = \frac{\Omega}{h}$ and $\frac{dK}{dt} = [\Omega - hK]$. It follows that

$$\frac{dL_*}{dt} = -\frac{1}{h} [\Omega - hK]^2. \quad (15)$$

$\frac{dL_*}{dt} < 0$ is a strictly Lyapunov function as presented in eq. (15). Thus, the alcohol consumption point L_* is globally asymptotically stable for $\mathfrak{R}_0 > 1$ in the region Σ . From eq. (15), $\frac{dL_*}{dt} = 0$ if and only if we set $S_u = S_u^*, S_a = S_a^*, E = E^*, I_a = I_a^*, I_d = I_d^*$, and $R = R^*$ then, $\frac{dL_*}{dt}$ convergence in positive regions Σ as $t \rightarrow \infty$. ■

4. Numerical Results

Numerical simulations of the model are performed in this section to bolster the preceding section's findings. We implemented the Fourth-Order Runge–Kutta (RK-4) method to solve model (1), based on the parameter values in Table 1 and initial conditions

$$S_u = 50, S_a = 50, E = 10, I_a = 5, I_d = 5, R = 0. \quad (16)$$

Table 1. The parameter values of model (1)

Parameters	Values	Source
Ω	10^3	Assumed
a	0.1;0.5;0.9	Assumed
b	0.1;0.5;0.9	Assumed
k	0.1;0.5;0.9	Assumed
m	0.3	[8]
n	0.25	[8]
f	0.0027	[8]
g	0.25	[8]
h	0.05	Assumed
τ	0.13978	[8]
β	0.7	[8]
α	0.05	[8]
μ	0.7	[8]

Due to its accuracy, precision, and programming efficiency, the RK-4 Method is a preferred numerical approach.

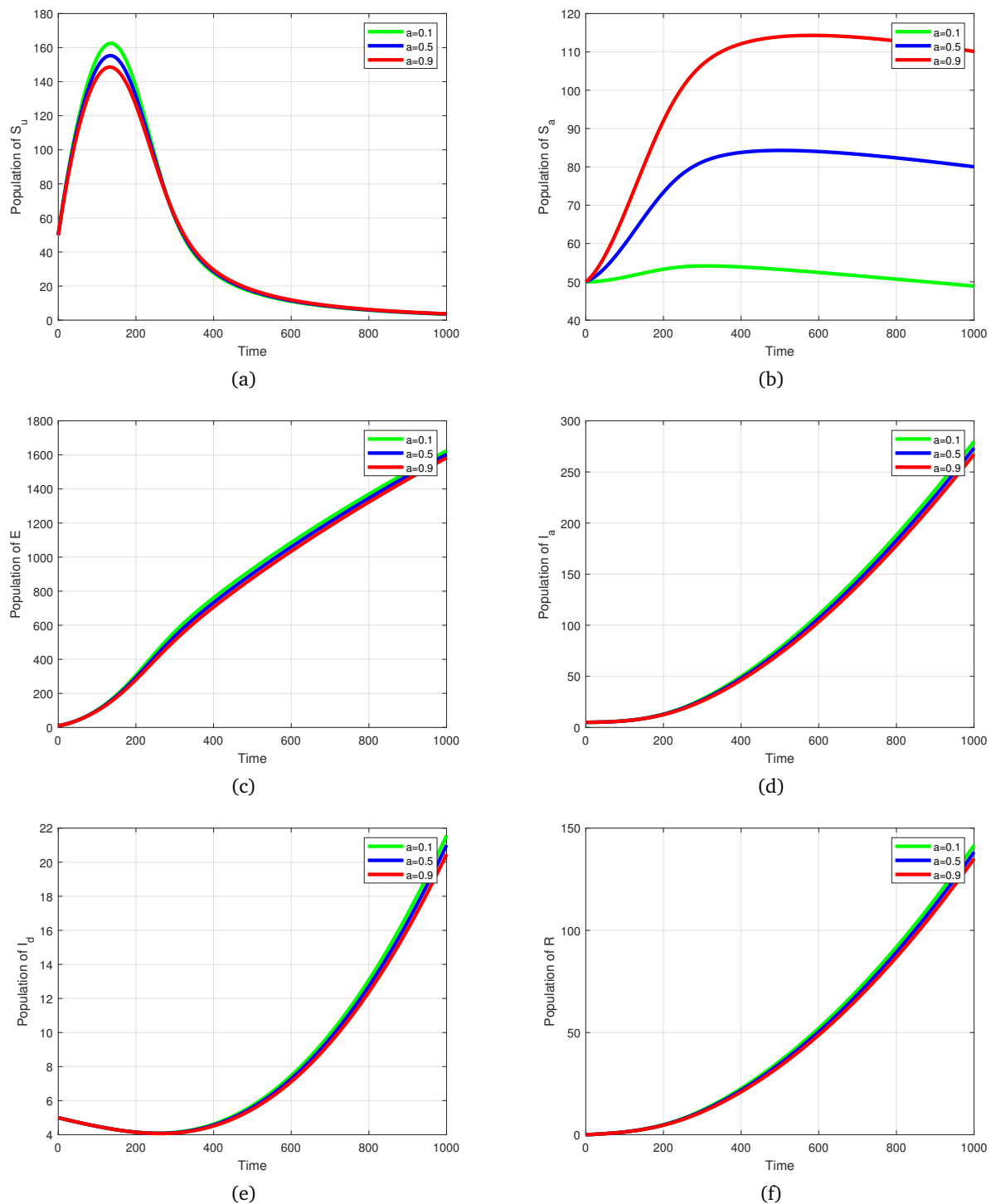


Figure 2. Simulation of the effects of education awareness parameter (a) with respect to time (t) for each compartments (a) S_u , (b) S_a , (c) E , (d) I_a , (e) I_d , and (f) R .

4.1. Effects of parameter a

We examine parameter a 's impact stemming from educational campaign policies in this subsection. Using $a = 0.1, 0.5, 0.9$, we consistently apply the parameter values found in Table 1 and the values of initial condition in (16). The population sizes of the S_u, E, I_a, I_d, R classes decreased due to the ex-

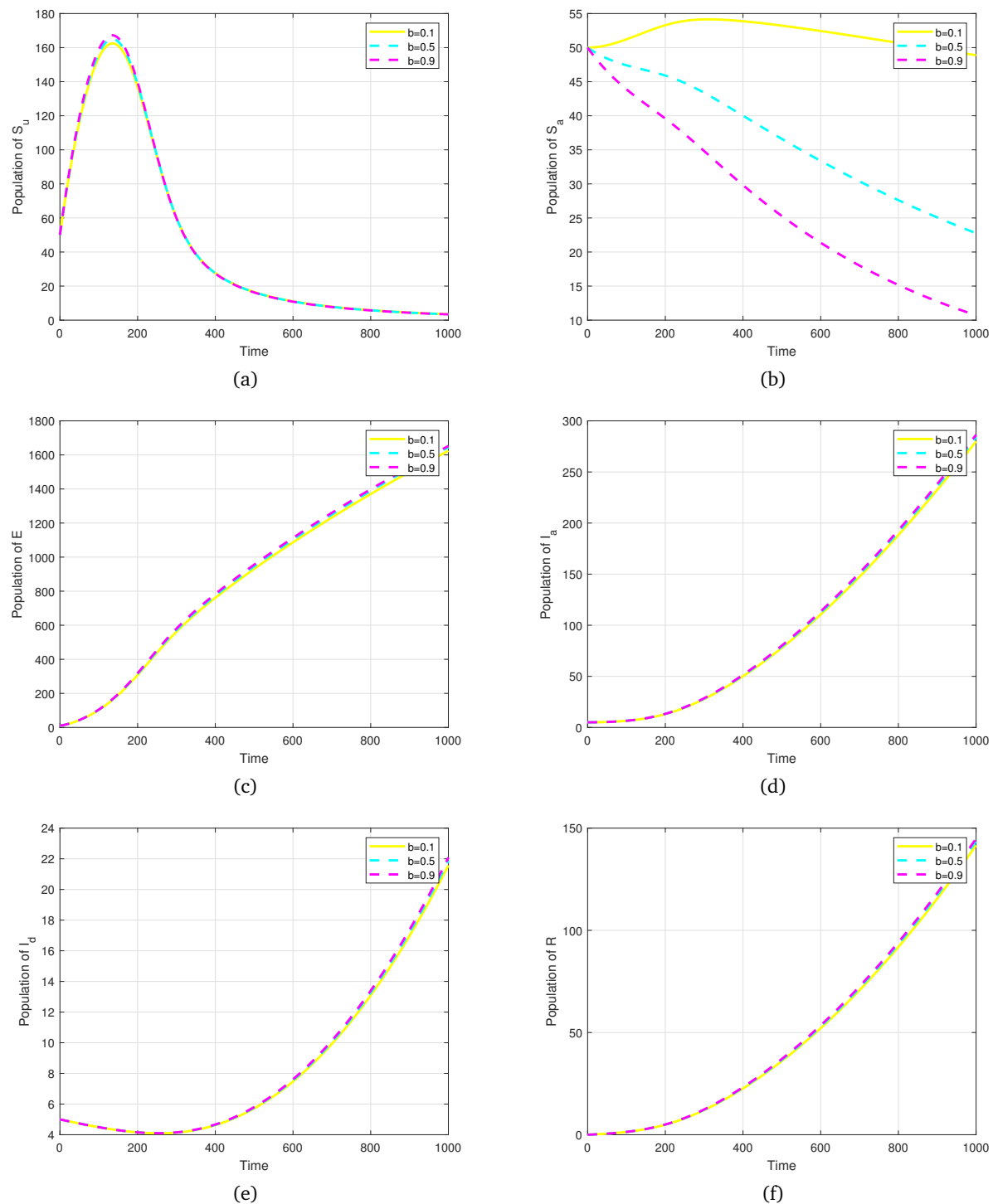


Figure 3. Simulation of the effects of unawareness parameter (b) with respect to time (t) for each compartments (a) S_u , (b) S_a , (c) E , (d) I_a , (e) I_d , and (f) R .

panded awareness campaign. Reduced contact between asymptomatic (E) and unaware susceptible (S_u) individuals has led to an increase in the number of aware susceptible (S_a) individuals. Raising awareness helps reduce individual alcohol consumption. Details of this behavior are performed in Figure 2.

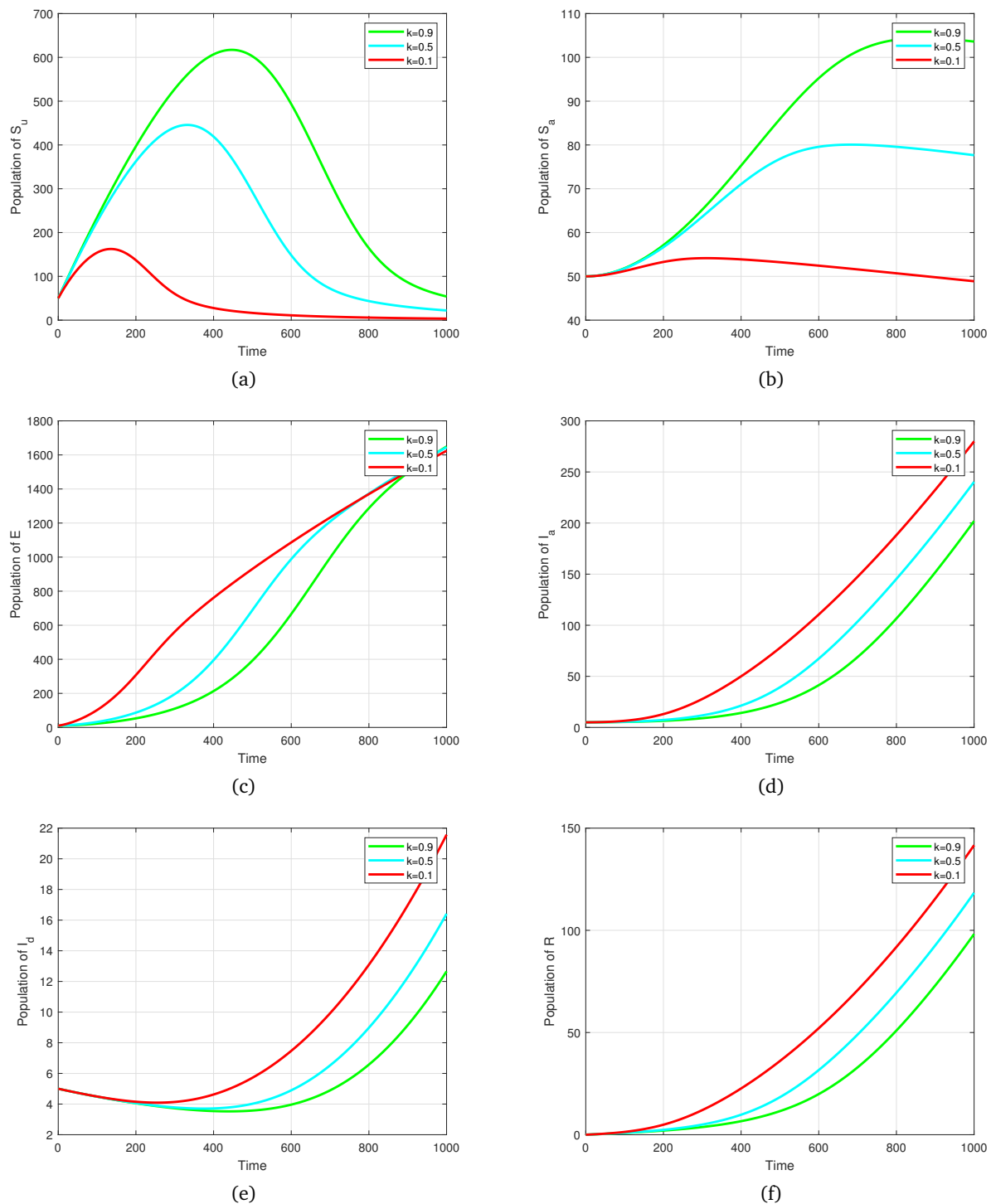


Figure 4. Simulation of the effects of unawareness parameter (b) with respect to time (t) for each compartments (a) S_u , (b) S_a , (c) E , (d) I_a , (e) I_d , and (f) R .

4.2. Effects of parameter b

We examine parameter b 's impact stemming from awareness vanishing in this subsection. Using $b = 0.1, 0.5, 0.9$, we consistently apply the values found in Table 1 and the values of initial condition in (16). The population size of the S_u, E, I_a, I_d, R classes increased due to the expanded intervention ac-

tion. Increased contact between Addictive (I_a) and unaware susceptible (S_u) individuals has led to an increase in the number of Exposed (E), Addictive (I_a), depressed (I_d), and recovered (R) individuals. Raising awareness vanishing helps reduce individuals in the aware class. Details of this behavior are performed in Figure 3.

4.3. Effects of parameter k

We examine the impact of parameter k on the effectiveness of intervention policies. We chose k from $[0.1, 0.5, 0.9]$. Increased the value of intervention parameter (k) causing the peak of the Exposed (E), Addictive individuals (I_a), Depressed individuals (I_d), and recovered individuals (R) are decreases. Meanwhile, individuals of unaware susceptible and aware susceptible classes (S_u and S_a) increase. This shows that the policy to increase the intensity of intervention may reduce the spread of consumption of alcohol. The changes in the value of intervention parameters that affect the population of each class are presented in Figure 4.

5. Conclusion

A six-compartmental model involving a differential equations system has been launched to analyze the dynamics of the proposed alcohol consumption model. We have obtained the threshold parameter ($\mathfrak{R}_0$), through which the stability criterion of the system (1) at the equilibrium points are analyzed. It is observed that the system (1) is globally asymptotically stable at alcohol-free point (A_0) for $\mathfrak{R}_0 < 1$. Similarly, we have discovered that the system (1) is globally asymptotically stable conditionally at (A_*) for $\mathfrak{R}_0 > 1$. The high awareness vanishing parameter (b) may be linked to higher rates of alcohol consumption due to increased unawareness among individuals. Conversely, raising awareness (a) and intervention (k) levels can curb alcohol consumption.

Supplementary Information

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Conflict of interest. The authors declare no conflict of interest.

Data availability. All data presented are hypothetical and cited from prior publications.

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