Indonesian Journal of Computational and Applied Mathematics
https://ejournal.gammarisepub.com/index.php/indocam
Volume 1, Issue 1, pages 39–51, February 2025
https://doi.org/10.64182/indocam.v1i1.12, E-ISSN 3090-5281


INDONESIAN JOURNAL OF
COMPUTATIONAL
AND APPLIED
MATHEMATICS

Research Article

A Mathematical Model of the Mud Crab (*Scylla Sp.*) Involving Cannibalism and Refuge

Agung Sucipto Abubakar[1], **Hasan S. Panigoro**[2,✉], **Ismail Djakaria**[1], **Agusyarif Rezka Nuha**[2], **Isran K. Hasan**[1], **Asriadi**[1]

[1]Department of Mathematics, Universitas Negeri Gorontalo, Bone Bolango 96554, Indonesia
[2]Biomathematics Research Group, Universitas Negeri Gorontalo, Bone Bolango 96554, Indonesia

Abstract

Cannibalism, a common ecological phenomenon in various species, significantly impacts the population dynamics of mud crabs (Scylla sp.). This study develops a mathematical model to analyze the effects of cannibalism and protective refuges on the population sustainability of juvenile and adult mud crabs. The model identifies two equilibrium points: the extinction equilibrium and the coexistence equilibrium. Stability analysis using the Jacobian matrix reveals that the extinction equilibrium is locally asymptotically stable under specific conditions. In contrast, the coexistence equilibrium depends on the transition rate from juvenile to adult crabs and the effectiveness of protective measures. Numerical simulations demonstrate that increasing the transition rate and implementing higher levels of refuge protection mitigate the adverse effects of cannibalism, enhancing population stability. These findings provide a quantitative foundation for sustainable fisheries management and conservation strategies for mud crab populations in mangrove ecosystems.

Keywords : *Cannibalism · Refuge effect · Stability analysis · Numerical simulation · Mathematical modeling*
MSC2020 : *92D25 · 37N25 · 92B05 · 37M10 · 37M05*

1. Introduction

As a maritime nation, Indonesia has great potential in fisheries resources, including mud crabs (*Scylla Sp.*). These crabs play a crucial role in maintaining the health of mangrove ecosystems and hold substantial economic value, especially as an export commodity. With a high nutritional profile consisting of 47.5% protein and 11.2% fat, mud crabs are considered a valuable food source [1]. In 2023, Indonesia exported 1.7 tons of mangrove crabs from Maluku to ASEAN countries, representing a 71% increase compared to the previous year [2]. Beyond their economic value, the waste generated from mud crab shells holds potential for conversion into biodiesel, offering a promising alternative energy source [3].

The mud crab (*Scylla sp.*), a species from the Portunidae family, is widely distributed across the Indo-West Pacific region [4]. These crabs inhabit mangrove ecosystems, including tropical estuaries, rivers, and tidal areas. Their presence contributes significantly to the livelihoods of communities residing near mangrove forests [5–8]. Mangrove ecosystems provide habitat and feeding grounds for crabs and offer protection. The presence of mangroves is crucial for maintaining crab populations, as these crabs are particularly vulnerable to predation by other predators, such as monitor lizards, and cannibalism among their species [9–11].

✉ Corresponding author. Email: hspanigoro@ung.ac.id
received: 20 February 2025, revised: 22 February 2025, accepted: 26 February 2025, published Online: 28 February 2025

GAMMARISE PUBLISHING

Cannibalism refers to consuming part or all of an individual of the same species [12–14]. In mud crabs, cannibalism is a common occurrence, mainly when there are differences in body size and during the molting process [15]. Crabs have softer exoskeletons during molting, making them more vulnerable to predation by other crabs [16]. The availability of shelter plays a crucial role in reducing the incidence of cannibalism, as sufficient shelter can significantly minimize this behavior among crabs [17, 18].

Mathematical approaches using deterministic models based on differential equations are commonly employed to study population dynamics [19–21]. These methods help to analyze the effects of cannibalism, allowing for the development of strategies to maintain ecosystem balance [21]. Previous research, such as that conducted by Li et al. [13], has investigated the impact of cannibalism on prey-predator models with logistic prey population growth. Similarly, Wilda et al. [22] explored prey-prey interaction models categorized by age, utilizing a Monod-Haldane response function and incorporating cannibalism into the interactions.

This research investigates the population dynamics of mud crabs exhibiting cannibalistic behavior through a mathematical approach. The decline in mud crab populations is driven by predation and cannibalism, but this risk can be mitigated by protective measures such as shelter [18]. Consequently, this study incorporates the protection factor to assess its role in minimizing cannibalism. The developed differential equation model is analyzed to examine the equilibrium point's behavior, aiming to determine its stability. Finally, numerical simulations are performed to interpret the population dynamics of mud crabs, considering the effects of cannibalism and refuge factors.

2. Methods

The method employed in this research is a literature review that involves tracing sources from scientific journals, articles, books, and other relevant references about mangrove crabs and mathematical modeling. The research process includes the following steps: (1) problem determination, (2) problem formulation, (3) literature review, (4) formulation of assumptions, (5) model development, (6) model validation, (7) analysis, (8) numerical simulations, and (9) biological interpretation and conclusion.

In analyzing and solving the problem, several main steps of the search were carried out, as follows: (1) the formulation of a mathematical model incorporating cannibalism within the mangrove crab (*Scylla Sp.*) population, based on established assumptions; (2) identification of the equilibrium points of the constructed model; (3) analysis of the stability of these equilibrium points through linearization, utilizing Jacobian matrices; (4) execution of numerical simulations using *Spyder 5.1.5* software with the *Python 3.9* programming language; and (5) provision of biological interpretations of the numerical simulation results obtained.

3. Results and Discussion

3.1. Model Formulation

In this model, the mangrove crab population is divided into juvenile and adult crabs. The model is then built based on the following assumptions. In mud crab ecosystems, populations are categorized into juvenile mud crabs (u) and adult mud crabs (v). The intrinsic growth rate of juvenile crabs is proportional to the population density of adult crabs and is denoted by rv. Juvenile crabs transition into adults at a rate represented by γ. Competition for food is characterized by the parameter η. Additionally, cannibalism occurs, with adult crabs preying on juvenile crabs, increasing the juvenile population by n while simultaneously decreasing it by m. The protection factor for juvenile crabs against adult cannibalism is denoted by θ, reducing the probability of predation to $1-\theta$. Furthermore, natural mortality contributes to population decline and is represented by μ.

After that, a food chain diagram is constructed based on the assumptions above, as illustrated in Figure 1.

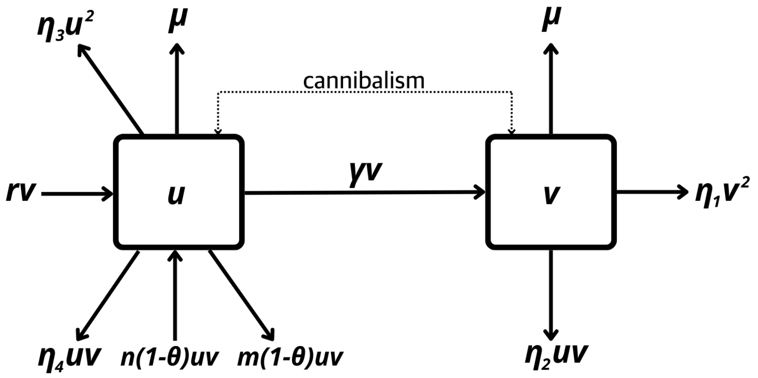


Figure 1. Interaction model diagram of juvenile and adult mud crabs

Based on the food chain diagram presented above, a model in the form of a system of equations is formulated as follows.

$$\begin{aligned}\frac{du}{dt} &= rv + (n-m)(1-\theta)uv - \eta_3 u^2 - \eta_4 uv - \gamma u - \mu u,\\ \frac{dv}{dt} &= \gamma u - \eta_1 v^2 - \eta_2 uv - \mu v.\end{aligned} \tag{1}$$

The parameters used in model (1) are detailed in Table 1 below.

Table 1. Biological parameter interpretation of model (1)

Parameter	Description
u	Total population of juvenile mud crabs
v	Total population of adult mud crabs
θ	Constant protection of juvenile mud crabs from cannibalism
r	Intrinsic growth rate of juvenile mud crabs
γ	Transition rate of juvenile mud crabs to adults
η_1	The rate of competition between juvenile mud crabs and their neighbors
η_2	The rate of competition between juvenile mud crabs and adult mud crabs occurring in the adult mud crab population
η_3	The rate of competition between adult mud crabs and each other
η_4	The rate of competition between juvenile mud crabs and adult mud crabs that occurs in the juvenile mud crab population
m	The rate of decline in the juvenile mud crab population due to cannibalism by adult mud crabs
n	rate of increase of juvenile mud crab population due to cannibalism by adult mud crabs
μ	Natural mortality rate of both populations

3.2. Equilibrium Point

The equilibrium points of the system of Equation (1) can be determined by solving the following conditions.

$$\frac{du}{dt} = 0, \quad \frac{dv}{dt} = 0.$$

These can be expressed as

$$\begin{aligned} rv + (n-m)(1-\theta)uv - \eta_3 u^2 - \eta_4 uv - \gamma u - \mu u &= 0,\\ \gamma u - \eta_1 v^2 - \eta_2 uv - \mu v &= 0.\end{aligned} \tag{2}$$

The equilibrium points are obtained by identifying the positive solutions of eq. (2). Consequently, two equilibrium points are provided as follows.

1. Extinction point of the entire population $E_0 = (0,0)$
2. Existence point of both populations $E^* = (u^*, v^*)$ with $u^* = \dfrac{v^*(\eta_1 v^* + \mu)}{\gamma - \eta_2 v^*}$ and v^* is the solution of the following polynomial equation.

$$\begin{aligned}&\left[\eta_1\left(\eta_1\eta_3 - \eta_2\left(\eta_4 - m\theta + m + n(\theta-1)\right)\right)\right]v^3 + \left[r\eta_2^2 - 2\eta_1\eta_3\mu + \mu\eta_2\eta_4 - \eta_1\eta_4\gamma + \eta_1\eta_2\gamma\right.\\ &\left.+\mu\eta_1\eta_2 + m(\theta-1)(\eta_1\eta_2 - \eta_2\mu) - n(\theta-1)(\eta_1\gamma - \eta_2\mu)\right]v^2 - \left[2r\eta_2\gamma + \mu^2(\eta_3 - \eta_2\right.\\ &\left.-\gamma\mu\left(\eta_1 - \eta_2 + \eta_4 + m - m\theta + n(\theta-1)\right) - \eta_1\gamma^2\right]v + \gamma\big(r\gamma - \mu(\gamma+\mu)\big) = 0\end{aligned} \tag{3}$$

Suppose v^* is a positive real root of eq. (3), which can be expressed as follows

$$v^3 + 3a_1v^2 + 3a_2v + a_3 = 0, \tag{4}$$

where

$$\begin{aligned}3a_1 &= \frac{\begin{array}{c}\eta_2^2 - 2\eta_1\eta_3\mu + \mu\eta_2\eta_4 - \eta_1\eta_4\gamma + \eta_1\eta_2\gamma + \mu\eta_1\eta_2\\ +m(\theta-1)(\eta_1\eta_2 - \eta_2\mu) - n(\theta-1)(\eta_1\gamma - \eta_2\mu)\end{array}}{\eta_1\left(\eta_1\eta_3 - \eta_2\left(\eta_4 - m\theta + m + n(\theta-1)\right)\right)}\\ 3a_2 &= \frac{2r\eta_2\gamma + \mu^2(\eta_3 - \eta_2 - \gamma\mu\left(\eta_1 - \eta_2 + \eta_4 + m - m\theta + n(\theta-1)\right) - \eta_1\gamma^2}{eta_1\left(\eta_1\eta_3 - \eta_2\left(\eta_4 - m\theta + m + n(\theta-1)\right)\right)}\\ a_3 &= \frac{\gamma\big(r\gamma - \mu(\gamma+\mu)\big)}{\eta_1\left(\eta_1\eta_3 - \eta_2\left(\eta_4 - m\theta + m + n(\theta-1)\right)\right)}\end{aligned}$$

By transforming $z = v + a_1$, eq. (4) can be simplified into

$$h(z) = z^3 + 3rz + s = 0, \tag{5}$$

where

$$\begin{aligned}r &= a_2 - a_1^2,\\ s &= a_3 - 3a_1a_2 - 2a_1^3.\end{aligned}$$

Furthermore, to define the point v we refer to the criteria outlined in [23], which are as follows.

(i) Equation (5) has one positive root if $s < 0$, or
(ii) If $s > 0$ and $r < 0$, then
  (ii.a) Equation (5) has one positive root of multiplicity two if $s^2 + 4r^3 = 0$, or
  (ii.b) Equation (5) has two positive roots if $s^2 + 4r^3 < 0$, or
(iii) Equation (5) has a unique positive root if $s = 0$ and $r < 0$.

Both equilibrium points above are biological equilibrium points, which means they always exist and are defined on $\mathbb{R}_2^+$.

3.3. *Stability Analysis*

In this section, the dynamics of the solution are studied by analyzing each equilibrium point, followed by linearization of the system using the Jacobian matrix. Accordingly, the Jacobian matrix for Equation (1) is derived as follows.

$$J(u,v) = \begin{bmatrix}(n-m)(1-\theta)v - 2\eta_3u - \eta_4v - (\gamma+\mu) & r + (n-m)(1-\theta)u - \eta_4u\\ \gamma - \eta_2v & -2\eta_1v - \eta_2u - \mu\end{bmatrix}. \tag{6}$$

The matrix $J(u,v)$ represents the Jacobian matrix, which is subsequently used to analyze the stability of the equilibrium points.

3.3. Stability of Equilibrium Point E_0

By subtitusing $E_0 = (0,0)$, into the jacobian matrix (6) we obtain

$$J(E_0) = \begin{bmatrix} -(\gamma+\mu) & r \\ \gamma & -\mu \end{bmatrix}.$$

Furthermore, the stability properties of the equilibrium point $E_0 = (0,0)$ are described in Theorem 1 as follows.

Theorem 1. *The equilibrium point* $E_0 = (0,0)$ *is locally asymptotically stable if* $r < \dfrac{\mu(\gamma+\mu)}{\gamma}$.

Proof. By solving $det(J(E_0 - \lambda I)) = 0$, we obtain the following characteristic equation

$$\lambda^2 + (\gamma+2\mu)\lambda + \mu^2 + \gamma\mu - r\mu = 0.$$

Solving this characteristic equation yields two eigenvalues:

$$\lambda_1 = \frac{1}{2}\left(-(\gamma+2\mu) - \sqrt{\gamma^2+4r\gamma}\right),$$
$$\lambda_2 = \frac{1}{2}\left(-(\gamma+2\mu) + \sqrt{\gamma^2+4r\gamma}\right).$$

Since $\gamma, \mu, r > 0$, it follows that $\lambda_1 < 0$. Further analysis is conducted on λ_2,

a. For the case $\lambda_2 < 0$

$$\begin{aligned}
&\lambda_2 > 0\\
&\frac{1}{2}\left(-(\gamma+2\mu) + \sqrt{\gamma^2+4r\gamma}\right) > 0\\
&\sqrt{\gamma^2+4r\gamma} > \gamma+2\mu\\
&\left(\sqrt{\gamma^2+4r\gamma}\right)^2 > (\gamma+2\mu)^2\\
&\gamma^2+4\gamma r > \gamma^2+4\gamma\mu+4\mu^2\\
&4\gamma r > 4\gamma\mu+4\mu^2\\
&\gamma r > \gamma\mu+\mu^2\\
&r > \frac{(\mu\gamma+\mu^2)}{\gamma}
\end{aligned}$$

b. For the case $\lambda_2 > 0$

$$\begin{aligned}
&\lambda_2 < 0\\
&\frac{1}{2}\left(-(\gamma+2\mu) + \sqrt{\gamma^2+4r\gamma}\right) < 0\\
&r < \frac{\gamma\mu+\mu^2}{\gamma}
\end{aligned}$$

Therefore, if the condition $r > \dfrac{\mu(\gamma+\mu)}{\gamma}$ is satisfied, the equilibrium point is unstable. Conversely, if $r < \dfrac{\mu(\gamma+\mu)}{\gamma}$, the equilibrium point $E_0 = (0,0)$ is locally asymptotically stable. ∎

3.3. *Stability of Equilibrium Point E^**

The stability properties of the equilibrium point $E^* = (u^*, v^*)$ are given in Theorem 2 below.

Theorem 2. *The equilibrium point $E^* = (u^*, v^*)$ is locally asymptotically stable if $\gamma > \gamma_a$ and $\gamma < \gamma_b$, with*

$$\begin{aligned}
\gamma_a &= (n-m)(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - 2\eta_1 v^* - \eta_2 u^* - 2\mu \\
\gamma_b &= \frac{\left[\left((n-m)(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - \mu\right)(2\eta_1 v^* + \eta_2 u^* + \mu)\right] - \eta_2 v^*(r + (n-m)(1-\theta)u^* - \eta_4 u^*)}{2\eta_1 v^* + \eta_2 u^* + \mu - r - (n-m)(1-\theta)u^* + \eta_4 u^*}
\end{aligned}$$

Proof. By substituting $E^* = (u^*, v^*)$ nto the Jacobian matrix (6), the following matrix is obtained.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

where

$$\begin{aligned}
A_{11} &= (n-m)(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - (\gamma + \mu), \\
A_{12} &= r + (n-m)(1-\theta)u^* - \eta_4 u^*, \\
A_{21} &= \gamma - \eta_2 v^*, \\
A_{22} &= -2\eta_1 v^* - \eta_2 u^* - \mu.
\end{aligned}$$

By solving the characteristic equation of the Jacobian matrix, we obtain:

$$\lambda^2 - (A_{11} + A_{22})\lambda + A_{11}A_{22} - A_{12}A_{21} = 0.$$

or equivalently,

$$\lambda^2 + a_1\lambda + a_2 = 0,$$

where

$$\begin{aligned}
a_1 &= -(A_{11} + A_{22}), \\
a_2 &= A_{11}A_{22} - A_{12}A_{21}.
\end{aligned}$$

According to the Routh-Hurwitz theorem [24], the equilibrium point $E^* = (u^*, v^*)$ is locally asymptotically stable if the conditions $a_1 > 0$ and $a_2 > 0$ are satisfied. The conditions under which $a_1 > 0$ and $a_2 > 0$ are explained as follows..

1. Since $a_1 > 0$, we obtain

$$\begin{aligned}
a_1 &> 0 \\
-(A_{11} + A_{22}) &> 0 \\
-\left[\left((n-m)(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - \gamma - \mu\right) + (-2\eta_1 v^* - \eta_2 u^* + \mu)\right] &> 0 \\
-(n-m)(1-\theta)v^* + 2\eta_3 u^* + \eta_4 v^* + \gamma + 2\eta_1 v^* + \eta_2 u^* + 2\mu &> 0 \\
\gamma &> (n-m)(1-\theta)v^* - 2\eta_3 u^* \\
&\quad - \eta_4 v^* - 2\eta_1 v^* - \eta_2 u^* - 2\mu \\
\gamma &> \gamma_a
\end{aligned}$$

2. Since $a_2 > 0$, we have

$$\begin{aligned}
a_2 &> 0 \\
A_{11}A_{22} &> A_{12}A_{21} \\
\big[((n-m))(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - \gamma - \mu\big) & \\
(-2\eta_1 v^* - \eta_2 u^* - \mu)\big] &> \big(r + (n-m)(1-\theta)u^* - \eta_4 u^*\big) \\
&\quad (\gamma - \eta_2 v^*) \\
-\gamma(r + (n-m)(1-\theta)u^* - \eta_4 u^*) &> \big[\big((n-m)(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - \mu\big) \\
&\quad (2\eta_1 v^* + 2\eta_2 u^* + \mu)\big] \\
&\quad - \eta_2 v^*(r + (n-m)(1-\theta)u^* - \eta_4 u^*) \\
\gamma\big[2\eta_1 v^* + \eta_2 u^* + \mu - r & \\
-(n-m)(1-\theta)u^* - \eta_4 u^*\big] &> \big[\big((n-m)(1-\theta)v^* - 2\eta_3 u^* - \eta_4 v^* - \mu\big) \\
&\quad (2\eta_1 v^* + 2\eta_2 u^* + \mu)\big] \\
&\quad - \eta_2 v^*(r + (n-m)(1-\theta)u^* - \eta_4 u^*) \\
\gamma &< \gamma_b
\end{aligned}$$

■

3.4. Numerical Simulation

In this section, we numerically investigate the dynamics of the model (1) using *Python 3.9*. Simulations are performed to gain insights into the interaction dynamics between the two species. The analysis also includes varying specific parameter values, such as the transition rate from juvenile to adult mud crabs and the protection constant. The parameter values are provided in Table 2 below.

Table 2. Parameter Values

No.	Simulation 1	Simulation 2	Source
r	0.5	1	assumption
m	0.2	0.2	assumption
n	0.1	0.1	assumption
γ	0.1	0.15	assumption
θ	0.3	0.3	[14]
η_1	0.1	0.1	assumption
η_2	0.1	0.1	assumption
η_3	0.1	0.1	assumption
η_4	0.1	0.1	assumption
μ	0.25	0.25	assumption

The first simulation begins by selecting parameter values from Simulation 1 in Table 2. Based on the stability conditions outlined in Theorem 1, it can be concluded that point E_0 is locally asymptotically stable. This is illustrated in the phase portrait in Figure 2 and the time-series plot in Figure 3. By choosing initial conditions relatively close to E_0, it is observed that the solution converges towards equilibrium point E_0, indicating that with these parameter values, the solution approaches the species' extinction point. Furthermore, the parameter values for point E_0 in Simulation 1 (Table 2) align with the stability conditions in Theorem 1, where $r < \dfrac{\mu(\gamma+\mu)}{\gamma}$ holds with $r = 0.5$ and $\dfrac{\mu(\gamma+\mu)}{\gamma} = 0.875$.

In the second figure above, it can be seen that by selecting some initial value conditions that are relatively close to E_0, the solution appears to move towards the equilibrium point E_0, which means that using these parameter values will lead to the species' extinction point.

Next, the simulation is further conducted concerning the stability conditions of the equilibrium point E^*. For this, the previous parameter values are updated to the new values in Simulation 2 in

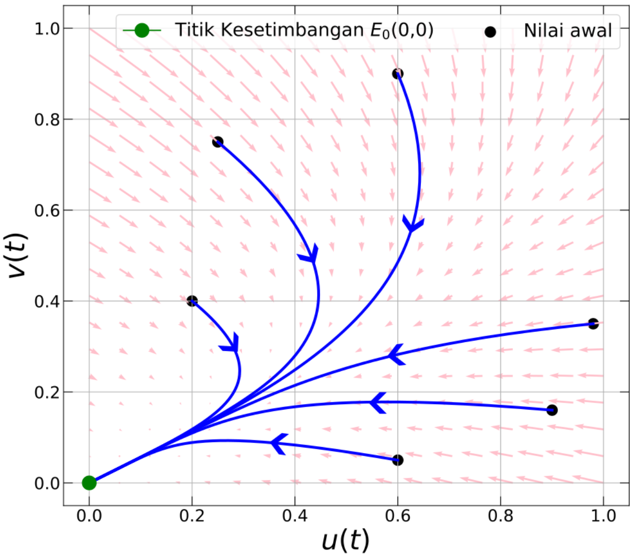


Figure 2. Phase portrait around Equilibrium point E_0

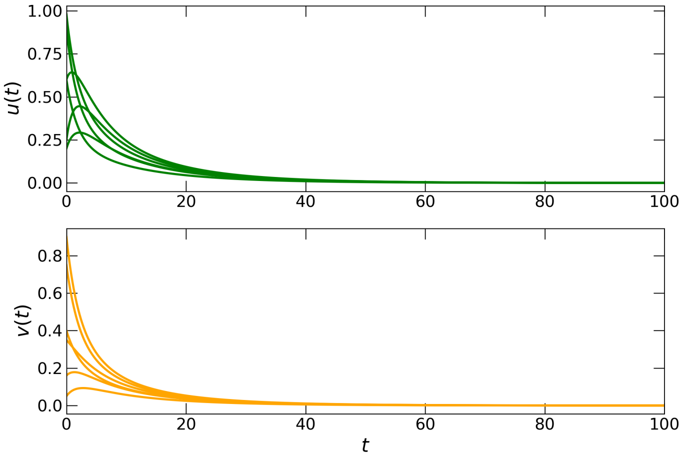


Figure 3. Time Series plot for Equilibrium point E_0

Table 2. Based on these parameter values, Simulation 2 corresponds to Theorem 2, where $\gamma > \gamma_a$ or $0.15 > -0.705$ and $\gamma < \gamma_b$ or $0.15 < 0.190$. By choosing specific initial conditions, the phase portrait and the *time-series* plot are presented in Figure 4 and Figure 5.

The two figures show that the solution initially moves to point E_0 as in Figure 3. After changing some parameter values, the solution moves closer and towards point E^* over time, which is the point of existence of the species. This shows that using the parameter values in simulation 2, a solution will be obtained that shows the existence of juvenile and adult Scylla serrata and both populations that will not lead to extinction.

3.4. Impact of changing the transition rate of juvenile to adult mud crabs (γ)

The simulation in this section is carried out with the same calculation but by choosing different values of γ and keeping the other parameter values the same as the parameter values in simulation 2 in Table 2.

Choosing the values of $\gamma = 0.095$, $\gamma = 0.145$, and $\gamma = 0.175$ resulted in the system stabilizing at

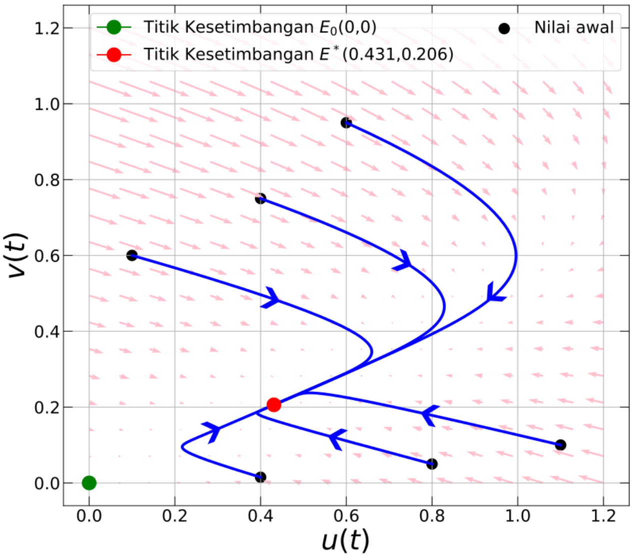


Figure 4. Phase portrait around Equilibrium point E^*

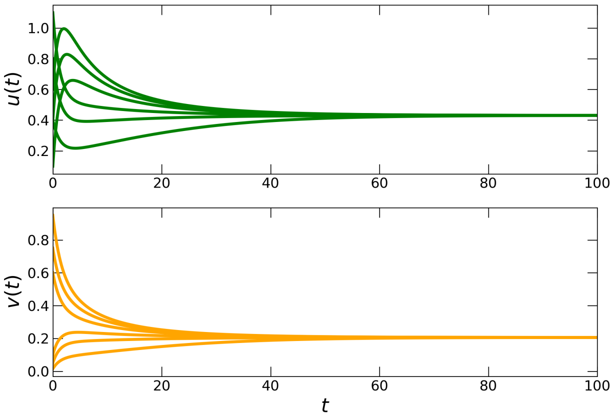


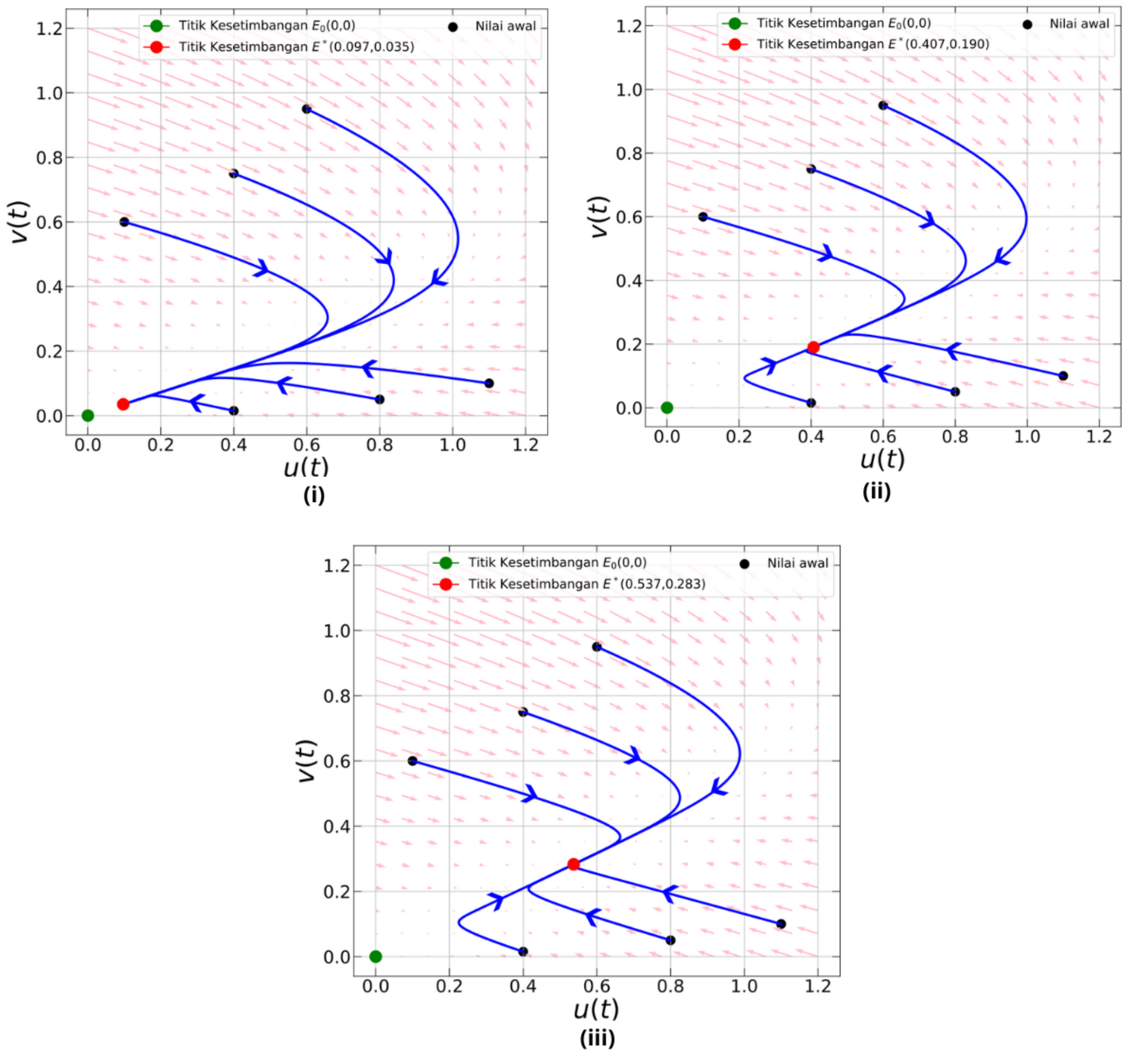
Figure 5. Time Series plot for Equilibrium point E_0

the equilibrium point E^*, with the number of individuals in the juvenile mud crab population with $\gamma = 0.095$ stabilizing at 0.097. In contrast, for the other two conditions of $\gamma = 0.145$ and $\gamma = 0.175$, the number of individuals in the juvenile mud crab population stabilized at 0.407 and 0.537, respectively. For the number of individuals in the adult mud crab population with a given value of γ, it will stabilize at 0.035 for $\gamma = 0.095$, and 0.190 and 0.283 for the other two values of γ. The simulation results based on the explanation above can be seen in the following figure.

Furthermore, it can be seen in Figure 7 that the higher the value of γ given, the higher the rate of increase of individuals, both from juvenile and adult mud crabs. The same thing happens to the solution condition, where this simulation uses the parameters from simulation two, which shows that the higher the value of γ given, the faster the solution to the equilibrium point E^*.

3.4. Impact of Variations in the Protection Level (θ) on the Dynamics of the Solution

In this section, the simulation aims to see the change in protection level (θ) on the dynamics of the given system. Still using the same parameter values used in simulation 2 in Table 2 but with different values of θ. The θ values in question are $\theta = 0$, $\theta = 0.5$ and $\theta = 1$. By choosing these θ values,



(i) **(ii)** **(iii)**

Figure 6. Population dynamics for **(i)** $\gamma = 0.095$, **(ii)** $\gamma = 0.145$ dan **(iii)** $\gamma = 0.175$

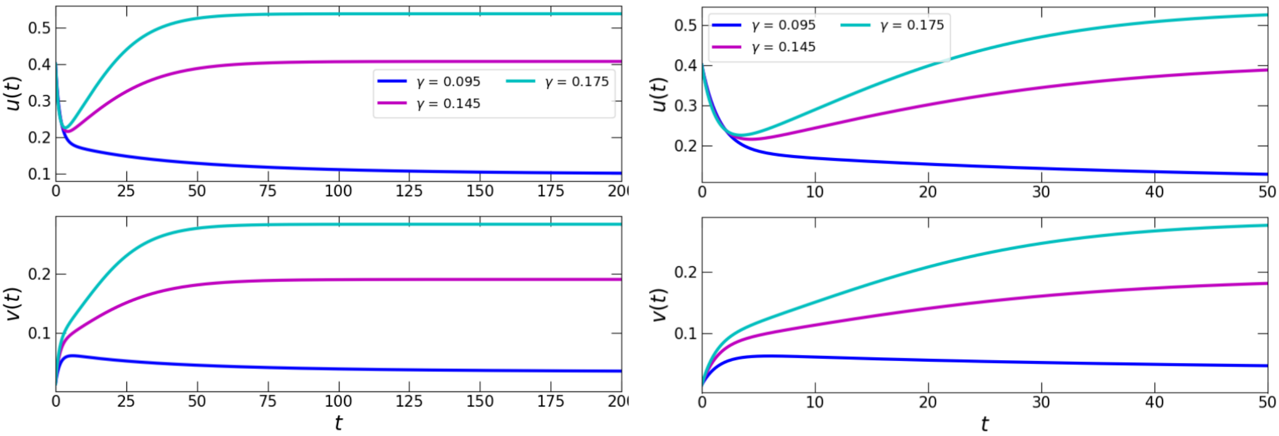
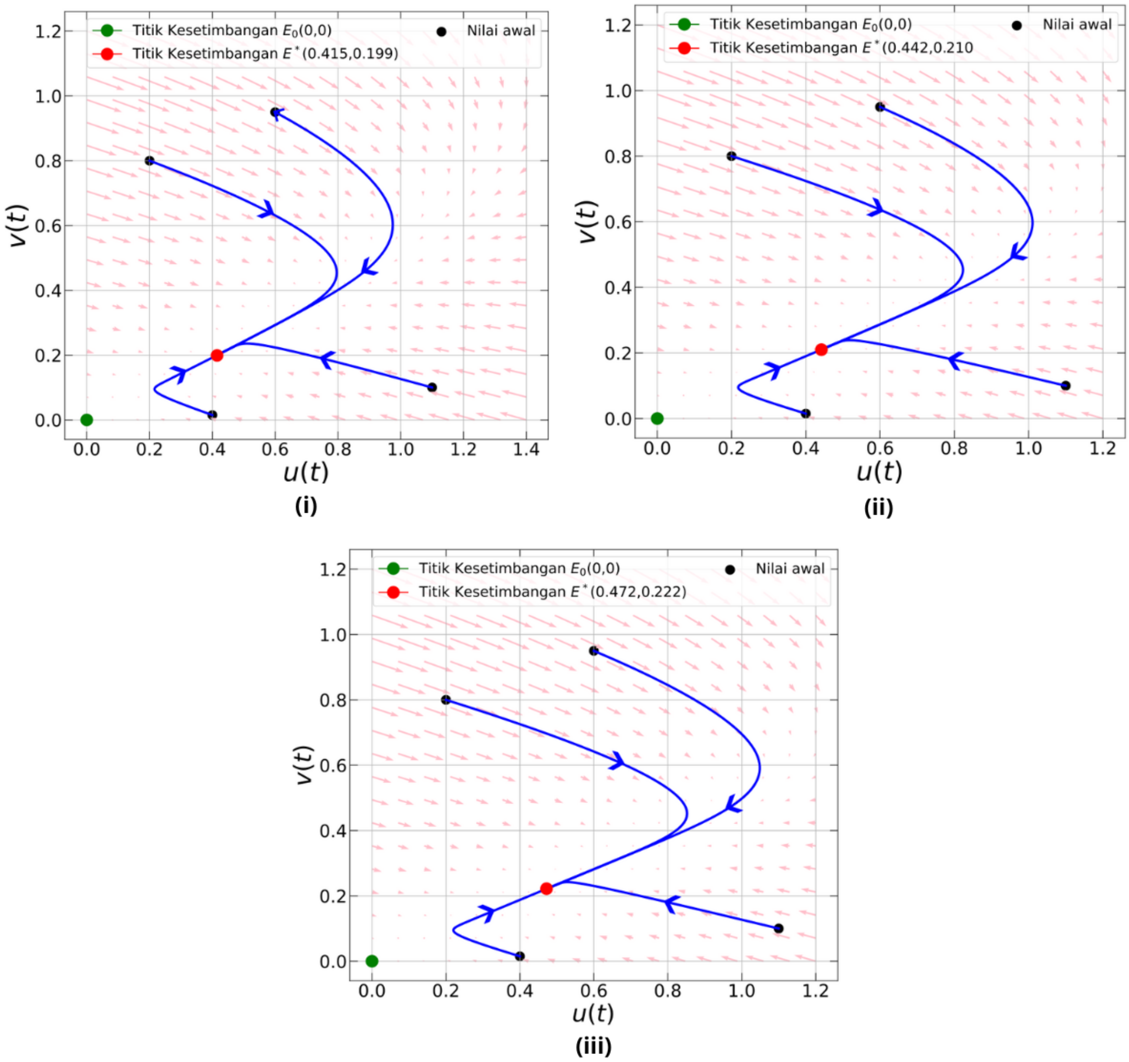


Figure 7. Time Series plot for changes in γ value

the system will show stability at the equilibrium point E^* as shown in Figure 8. With the number of individuals in the juvenile mud crab population with $\theta = 0$ will be stable at 0.415, $\theta = 0.5$ the number of individuals will be stable at 0.442 and $\theta = 1$ is stable at 0.472. Meanwhile, the number of individuals in the adult mud crab population with $\theta = 0$ will stabilize at 0.199, as well as 0.210 and 0.222 for the other θ values. Based on the explanation above, the following figure shows the simulation results.



(i)

(ii)

(iii)

Figure 8. Population dynamics for **(i)** $\theta = 0$, **(ii)** $\theta = 0.5$ dan **(iii)** $\theta = 1$

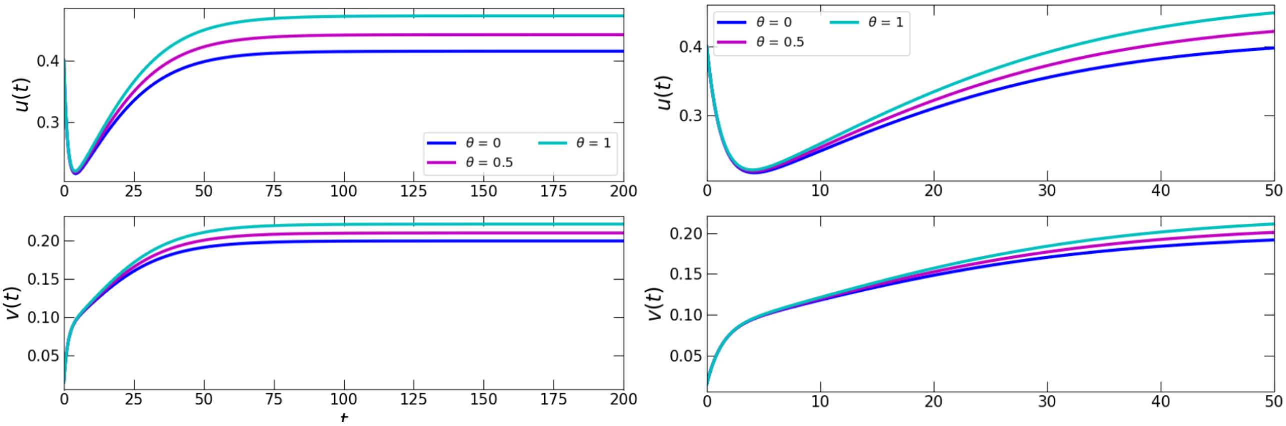


Figure 9. Time Series plot for change in θ value

Based on Figure 9, it can be seen from the dynamics shown by the time-series plot that the value of $\theta = 0$ or in the sense that there is no protection factor against cannibalism carried out results in a decrease in the number of individuals of both populations. In the opposite case, if the value of $\theta = 1$ or in the sense that there is complete protection against cannibalism is carried out, it will increase the number of individuals for both populations.

4. Conclusion

This study develops a mathematical model to analyze the population dynamics of mud crabs (*Scylla sp.*) considering the effects of cannibalism and protective refuges. The model identifies two equilibrium points: the extinction equilibrium (E_0) and the coexistence equilibrium (E^*). Stability analysis reveals that the extinction equilibrium is locally asymptotically stable when the juvenile growth rate is lower than the decline due to cannibalism. Conversely, the coexistence equilibrium can be maintained if the transition rate from juvenile to adult crabs and the level of protection are sufficiently high. Numerical simulations demonstrate that increasing the juvenile-to-adult transition rate (γ) and enhancing protection factors (θ) contribute to more stable population growth. When protection levels increase, the number of individuals in both population groups rises, mitigating the negative impacts of cannibalism. Therefore, the findings of this study provide a quantitative foundation for sustainable fisheries management and conservation strategies for mud crab populations in mangrove ecosystems.

Supplementary Information

Acknowledgements. We extend our heartfelt thanks to the editors and reviewers for their constructive comments and invaluable support in refining this work.

Funding. This research received no external funding.

Author Contributions. **Agung Sucipto Abubakar and Hasan S. Panigoro** Conceptualization, Methodology, Software, Formal Analysis, Investigation, Resources, Data Curation, Writing - Original Draft, Visualization, **Ismail Djakaria and Agusyarif Rezka Nuha:** Conceptualization, Methodology, Writing - Review & Editing, Supervision, Project Administration, Funding Acquisition. **Isran K. Hasan and Asriadi:** Conceptualization, Validation, Writing - Review & Editing, Supervision, Project Administration, Funding Acquisition. All authors read and approved the final manuscript.

Conflict of interest. The authors declare no conflict of interest.

Data availability. All data in this paper are hypothetically provided.

References

[1] Katiandagho B. Komposisi nutrien tubuh pada kepiting bakau (Scylla spp) yang diberi stimulan molting. Agrikan: Jurnal Agribisnis Perikanan. 2012;5(2):78-82. [Crossref].
[2] Hasan A. 1,7 Ton Kepiting Bakau Maluku Tembus Pasar ASEAN. Good News From Indonesia; 2023. [Website].
[3] Boey PL, Maniam GP, Hamid SA. Biodiesel production via transesterification of palm olein using waste mud crab (Scylla serrata) shell as a heterogeneous catalyst. Bioresource Technology. 2009;100(24):6362-8. [Crossref].
[4] Trivedi JN, Vachhrajani KD. Taxonomic account of genus Scylla (de Haan , 1833) from Gujarat State , India with two new records of species Taxonomic account of genus Scylla (de Haan , 1833) from Gujarat State , India with two new records of species. Arthropods. 2013;2(4). [Researchgate].
[5] Bir J, Islam SS, Sabbir W, Islam R, Huq KA. Ecology and reproductive biology of Mud Crab Scylla spp: A study of commercial mud crab in Bangladesh. International Journal of Academic Research and Development. 2020;5:1-7. [Website].
[6] Karniati R. Mangrove ecosystem in north sumatran (Indonesia) forests serves as a suitable habitat for mud crabs (scylla serrata and s. olivacea). Biodiversitas. 2021;22(3):1489-96. [Crossref].
[7] Mahbubi A. Developing a mud crab ecotourism business model in the mangrove forest ecosystems of Belitong UNESCO Global Geopark, Indonesia [Article]. Asian Journal of Forestry. 2025;9(1):13-23. [Crossref].
[8] Dewiyanti I. Some Biological Aspects of Mud Crabs (Scylla serrata) in Mangrove Ecosystem, Banda Aceh and Aceh Besar [Conference Paper]. BIO Web of Conferences. 2024;92. [Crossref].

[9] Hulathduwa Y. The effect of salinity on survival, bioenergetics and predation risk in the mud crabs Panopeus simpsoni and Eurypanopeus depressus [Article]. Marine Biology. 2007;152(2):363-70. [Crossref].

[10] Chen X. Intracohort cannibalism of the mud crab, Scylla paramamosain, megalopae: Mechanism, dynamics and adaptive significance [Book Chapter]. Crabs: Global Diversity, Behavior and Environmental Threats. 2014:75-90. [Website].

[11] Putri A, Bengen DG, Zamani NP, Salma U, Kusuma NP, Diningsih NT, et al. Mangrove Habitat Structure of Mud Crabs (Scylla serrata and S. olivacea) in the Bee Jay Bakau Resort Probolinggo, Indonesia. ILMU KELAUTAN: Indonesian Journal of Marine Sciences. 2022 jun;27(2):124-32. [Crossref].

[12] Zhang F, Chen Y, Li J. Dynamical analysis of a stage-structured predator-prey model with cannibalism. Mathematical Biosciences. 2019;307:33-41. [Crossref].

[13] Li J, Zhu X, Lin X, Li J. Impact of cannibalism on dynamics of a structured predator–prey system. Applied Mathematical Modelling. 2020;78:1-19. [Crossref].

[14] Rayungsari M, Suryanto A, Kusumawinahyu WM, Darti I. Dynamical Analysis of a Predator-Prey Model Incorporating Predator Cannibalism and Refuge. Axioms. 2022;11(3):116. [Crossref].

[15] Sanda T, Shimizu T, Dan S, Hamasaki K. Effect of body size on cannibalism in juvenile mud crab Scylla serrata (Decapoda: Brachyura: Portunidae) under laboratory conditions. Crustacean Research. 2021;50:87-93. [Crossref].

[16] Zi X, Li Y, Li G, Jia B, Jeppesen E, Zeng Q, et al. A molting chemical cue (N-acetylglucosamine-6-phosphate) contributes to cannibalism of Chinese mitten crab Eriocheir sinensis. Aquatic Toxicology. 2023;263:106666. [Crossref].

[17] Ly Van K, Arsa CP, Nguyen Thi NA, Tran Ngoc H. Use of different seaweeds as shelter in nursing mud crab, Scylla paramamosain : Effects on water quality, survival, and growth of crab. Journal of the World Aquaculture Society. 2022;53(2):485-99. [Crossref].

[18] Zhou D, Liu L, Huang X, Fang W, Fu Y, Li Y, et al. Effects of different shelters on feeding, molting, survival, and growth of Scylla paramamosain. Frontiers in Marine Science. 2023;10. [Crossref].

[19] Panigoro HS, Suryanto A, Kusumawinahyu WM, Darti I. Global stability of a fractional-order Gause-type predator-prey model with threshold harvesting policy in predator. Communications in Mathematical Biology and Neuroscience. 2021;2021(2021):63. [Crossref].

[20] Panigoro HS, Anggriani N, Rahmi E. Understanding the Role of Intraspecific Disease Transmission and Quarantine on the Dynamics of Eco-Epidemiological Fractional Order Model. Fractal and Fractional. 2023 aug;7(8):610. [Crossref].

[21] Ndii MZ. Pemodelan Matematika. Penerbit NEM; 2022. [Website].

[22] Wilda RW, Monita D, Afli F, Kadafi AM. Analisis Kestabilan Model Predator Prey dengan Tingkat Usia pada Dua Prey dan Dua Predator dengan Fungsi Respon Monod HalDane dan Kanibalisme. Epsilon: Jurnal Matematika Murni dan Terapan. 2023;17(2):249. [Crossref].

[23] Cai Y, Zhao C, Wang W, Wang J. Dynamics of a Leslie–Gower predator–prey model with additive Allee effect. Applied Mathematical Modelling. 2015;39(7):2092-106. [Crossref].

[24] Allen LJS. An Introduction to Mathematical Biology. Pearson/Prentice Hall; 2007. [Crossref].